Lifetimes of doubly charmed baryons

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(Received 25 September 2018; published 13 December 2018)

The lifetimes of doubly charmed hadrons are analyzed within the framework of the heavy quark expansion (HQE). Lifetime differences arise from spectator effects such as W-exchange and Pauli interference. The Ξ_{cc}^{++} baryon is longest-lived in the doubly charmed baryon system owing to the destructive Pauli interference absent in the Ξ_{cc}^{+} and Ω_{cc}^{+}. In the presence of dimension-seven contributions, its lifetime is reduced from $\sim 5.2\times 10^{-13}$ s to $\sim 3.0\times 10^{-13}$ s. The Ξ_{cc}^{+} baryon has the shortest lifetime of order 0.45×10^{-13} s due to a large contribution from the W-exchange box diagram. It is difficult to make a precise quantitative statement on the lifetime of Ω_{cc}^{+}. Contrary to Ξ_{cc} baryons, $\tau(\Omega_{cc}^{+})$ becomes longer in the presence of dimension-seven effects and the Pauli interference $\Gamma_{+}^{\rm int}$ even becomes negative. This implies that the subleading corrections are too large to justify the validity of the HQE. Demanding the rate $\Gamma_{+}^{\rm int}$ to be positive for a sensible HQE, we conjecture that the Ω_{c}^{0} lifetime lies in the range of $(0.75\sim 1.80)\times 10^{-13}$ s. The lifetime hierarchy pattern is $\tau(\Xi_{cc}^{++}) > \tau(\Omega_{cc}^{+}) > \tau(\Xi_{cc}^{+})$ and the lifetime ratio $\tau(\Xi_{cc}^{++})/\tau(\Xi_{cc}^{+})$ is predicted to be of order 6.7.

DOI: 10.1103/PhysRevD.98.113005

I. INTRODUCTION

Recently, the LHCb collaboration observed a resonance in the $\Lambda_c^+ K^- \pi^+ \pi^+$ mass spectrum at a mass of 3621.40 ± 0.78 MeV [1], which is consistent with expectations for the doubly charmed baryon Ξ_{cc}^{++} baryon. Subsequently, LHCb presented the first lifetime measurement of this charmed doubly baryon [2]

$$\tau(\Xi_{cc}^{++}) = (2.56^{+0.24}_{-0.22}\pm 0.14)\times 10^{-13}\ \text{s}. \qquad (1.1)$$

Theoretical predictions available in the literature [3–7] listed in Table I spread a large range, e.g., $\tau(\Xi_{cc}^{++})$ ranges from 0.2 to 1.6 ps. The lifetime hierarchy was predicted to be of the pattern $\tau(\Xi_{cc}^{++}) > \tau(\Omega_{cc}^{+}) > \tau(\Xi_{cc}^{+})$ in [4,5], but $\tau(\Xi_{cc}^{++}) > \tau(\Xi_{cc}^{+}) > \tau(\Omega_{cc}^{+})$ in [6].

In [8] we have shown that the heavy quark expansion (HQE) in $1/m_b$ works well for bottom hadrons. The calculated B meson lifetime ratios $\tau(B^+)/\tau(B_d^0)$ and $\tau(B_s^0)/\tau(B_d^0)$ in HQE are in excellent agreement with experiment, and the computed lifetime ratios $\tau(\Xi_b^-)/\tau(\Lambda_b^0)$, $\tau(\Xi_b^-)/\tau(\Xi_b^0)$ and $\tau(\Omega_b^-)/\tau(\Xi_b^-)$ also agree well with the data. On the contrary, the HQE to $1/m_c^3$ fails to give a satisfactory description of the lifetimes of both charmed mesons and charmed baryons. The HQE to order $1/m_c^3$ implies the lifetime hierarchy $\tau(\Xi_c^+) > \tau(\Lambda_c) > \tau(\Xi_c^0) > \tau(\Omega_c)$, which seems to be in agreement with the current one from the Particle Data Group (PDG) [9]. However, the quantitative estimates of charmed baryon lifetimes and their ratios are rather poor. For example, $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$ is calculated to be 1.03 [8], while experimentally it is measured to be 2.21 ± 0.15 [9]. Since the charm quark is not heavy, it is thus natural to consider the effects stemming from the next-order $1/m_c$ expansion. This calls for the subleading $1/m_Q$ corrections to spectator effects.

It turns out that the relevant dimension-seven spectator effects are in the right direction for explaining the large lifetime ratio of $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$, which is enhanced from 1.05 to 1.88, in better agreement with the experimental value [8]. However, the destructive $1/m_c$ corrections to $\Gamma(\Omega_c^0)$ are too large to justify the use of the HQE, namely, the predicted Pauli interference and semileptonic rates for the Ω_c^0 become negative, which certainly do not make sense. Demanding these rates to be positive for a sensible HQE, it has been conjectured in [8] that the Ω_c^0 lifetime lies in the range of $(2.3\sim 3.2)\times 10^{-13}$ s. This leads to the new lifetime pattern $\tau(\Xi_c^+) > \tau(\Omega_c^0) > \tau(\Lambda_c^+) > \tau(\Xi_c^0)$, contrary to the current hierarchy $\tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\Omega_c^0)$. This new charmed baryon lifetime pattern can be tested by LHCb.

Very recently, LHCb has reported a new measurement of the Ω_c^0 lifetime, $\tau(\Omega_c^0) = (2.68\pm 0.24\pm 0.10\pm 0.02)\times 10^{-13}$ s [10], using the semileptonic decay $\Omega_b^- \to \Omega_c^0 \mu^- \bar{\nu}_\mu X$ with $\Omega_c^0 \to pK^-K^-\pi^+$. This value is nearly four times larger than the current world-average value of

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TABLE I. Predicted lifetimes of doubly charmed baryons in units of 10^{-13} s.

	Kiselev et al. [3]	Kiselev et al. [4]	Guberina et al. [5]	Chang et al. [6]	Karliner et al. [7]
Ξ_{cc}^{++}	4.3 ± 1.1	4.6 ± 0.5	15.5	6.7	1.85
Ξ_{cc}^{+}	1.1 ± 0.3	1.6 ± 0.5	2.2	2.5	0.53
Ω_{cc}^{+}		2.7 ± 0.6	2.5	2.1	

$\tau(\Omega_c^0) = (0.69 \pm 0.12) \times 10^{-13}$ s [9] from fixed target experiments.[1] This indicates that the Ω_c^0, which is naively expected to be shortest-lived in the charmed baryon system owing to the large constructive Pauli interference, could live longer than the Λ_c^+ due to the suppression from $1/m_c$ corrections arising from dimension-seven four-quark operators.

In this work, we shall study the lifetimes of doubly charmed baryons within the framework of the HQE. It is organized as follows. In Sec. II, we give the general HQE expressions for inclusive nonleptonic and semileptonic widths. A special attention is paid to the doubly charmed baryon matrix elements of dimension-three and -five operators which are somewhat different from the ones of singly charmed baryons. We then proceed to discuss the relevant dimension-six and -seven four-quark operators. Evaluation of doubly charmed baryon matrix elements and numerical results are presented in Sec. III. Conclusions are given in Sec. IV.

II. THEORETICAL FRAMEWORK

Under the heavy quark expansion, the inclusive nonleptonic decay rate of a doubly heavy baryon $\mathcal{B}_{QQ}$ containing two heavy quarks QQ is given by [12,13]

$$\Gamma(\mathcal{B}_{QQ}) = \frac{1}{2m_{\mathcal{B}_{QQ}}} \mathrm{Im}\langle \mathcal{B}_{QQ}|T|\mathcal{B}_{QQ}\rangle = \frac{1}{2m_{\mathcal{B}_{QQ}}} \langle \mathcal{B}_{QQ}| \int d^4x T[\mathcal{L}_W^\dagger(x)\mathcal{L}_W(0)]|\mathcal{B}_{QQ}\rangle, \tag{2.1}$$

in analog to the case of a singly heavy hadron H_Q. Through the use of the operator product expansion, the transition operator T can be expressed in terms of local quark operators

$$\mathrm{Im}T = \frac{G_F^2 m_Q^5}{192\pi^3}\xi\left(c_{3,Q}\bar{Q}Q + \frac{c_{5,Q}}{m_Q^2}\bar{Q}\sigma\cdot GQ + \frac{c_{6,Q}}{m_Q^3}T_6 + \frac{c_{7,Q}}{m_Q^4}T_7 + \cdots\right), \tag{2.2}$$

where ξ is the relevant CKM matrix element, the dimension-six T_6 consists of the four-quark operators $(\bar{Q}\Gamma q)(\bar{q}\Gamma Q)$ with Γ representing a combination of the Lorentz and color matrices, while a subset of dimension-seven T_7 is governed by the four-quark operators containing derivative insertions. Hence,

$$\Gamma(\mathcal{B}_{QQ}) = \frac{G_F^2 m_Q^5}{192\pi^3}\xi\frac{1}{2m_{\mathcal{B}_{QQ}}}\left\{c_{3,Q}\langle \mathcal{B}_{QQ}|\bar{Q}Q|\mathcal{B}_{QQ}\rangle + \frac{c_{5,Q}}{m_Q^2}\langle \mathcal{B}_{QQ}|\bar{Q}\sigma\cdot GQ|\mathcal{B}_{QQ}\rangle + \frac{c_{6,Q}}{m_Q^3}\langle \mathcal{B}_{QQ}|T_6|\mathcal{B}_{QQ}\rangle + \frac{c_{7,Q}}{m_Q^4}\langle \mathcal{B}_{QQ}|T_7|\mathcal{B}_{QQ}\rangle + \cdots\right\}. \tag{2.3}$$

A. Dimension-three and -five operators

In heavy quark effective theory (HQET), the dimension-three operator $\bar{Q}Q$ in the rest frame has the expression

$$\bar{Q}Q = \bar{Q}\gamma_0 Q - \frac{\bar{Q}(i\vec{D})^2Q}{2m_Q^2} + \frac{\bar{Q}\sigma\cdot GQ}{4m_Q^2} + \mathcal{O}\left(\frac{1}{m_Q^3}\right), \tag{2.4}$$

with the normalization

$$\frac{\langle \mathcal{B}_{QQ}|\bar{Q}\gamma_0 Q|\mathcal{B}_{QQ}\rangle}{2m_{\mathcal{B}_{QQ}}} = 1. \tag{2.5}$$

Hence,

$$\frac{\langle \mathcal{B}_{QQ}|\bar{Q}Q|\mathcal{B}_{QQ}\rangle}{2m_{\mathcal{B}_{QQ}}} = 1 - \frac{\mu_\pi^2}{2m_Q^2} + \frac{\mu_G^2}{2m_Q^2} + \mathcal{O}\left(\frac{1}{m_Q^3}\right), \tag{2.6}$$

where

$$\mu_\pi^2 \equiv \frac{1}{2m_{\mathcal{B}_{QQ}}}\langle \mathcal{B}_{QQ}|\bar{Q}(i\vec{D})^2Q|\mathcal{B}_{QQ}\rangle = -\frac{1}{2m_{\mathcal{B}_{QQ}}}\langle \mathcal{B}_{QQ}|\bar{Q}(iD_\perp)^2Q|\mathcal{B}_{QQ}\rangle = -\lambda_1,$$
$$\mu_G^2 \equiv \frac{1}{2m_{\mathcal{B}_{QQ}}}\langle \mathcal{B}_{QQ}|\bar{Q}\frac{1}{2}\sigma\cdot GQ|\mathcal{B}_{QQ}\rangle = d_H\lambda_2. \tag{2.7}$$

The non-perturbative parameters λ_1 and λ_2 are independent of m_Q and have the same values for all particles in a given spin-flavor multiplet.

We first consider the non-perturbative parameter μ_π^2. In general, $\mu_\pi^2 = \langle p^2\rangle = \langle m_Q^2 v_Q^2\rangle$. The average kinetic energy of the diquark QQ and the light quark q is $T = \frac{1}{2}m_d v_d^2 + \frac{1}{2}m_q v_q^2$, where m_d (m_q) is the mass of the diquark (light quark). This together with the momentum conservation $m_d v_d = m_q v_q$ leads to

[1] An early conjecture of $\tau(\Omega_c^0)$ of order 2.3×10^{-13} s first presented in [11] by one of us is indeed consistent with the LHCb measurement.

$$v_d^2 = \frac{m_q T}{2m_Q^2 + m_Q m_q}. \qquad (2.8)$$

As shown in [3], the average kinetic energy T' of heavy quarks inside the diquark given by $\frac{1}{2} m_Q (v_{Q1}^2 + v_{Q2}^2)$ is equal to $T/2$ due to the color wave function of the diquark. Hence, the average velocity $\tilde{v}$ of the heavy quark inside the diquark is $\tilde{v}^2 = T/(2m_Q)$. The average velocity v_Q of the heavy quark inside the baryon $\mathcal{B}_{QQ}$ is [3]

$$v_Q^2 \approx \tilde{v}^2 + v_d^2 = \frac{T}{2m_Q} + \frac{m_q T}{2m_Q^2 + m_Q m_q}. \qquad (2.9)$$

Hence,

$$\mu_\pi^2(\mathcal{B}_{QQ}) \simeq m_Q \left(\frac{T}{2} + \frac{m_q T}{2m_Q + m_q} \right). \qquad (2.10)$$

We next turn to the parameter μ_G^2. In HQET, the mass of the singly heavy baryon $\mathcal{B}_Q$ has the expression

$$m_{\mathcal{B}_Q} = m_Q + \bar{\Lambda}_{\mathcal{B}_Q} + \frac{\mu_\pi^2}{2m_Q} - \frac{\mu_G^2}{2m_Q} + \mathcal{O}\left(\frac{1}{m_Q^2} \right), \qquad (2.11)$$

where $\bar{\Lambda}_{\mathcal{B}_Q}$ is a parameter of HQET and it can be regarded as the binding energy of the heavy hadron in the infinite mass limit. For the doubly heavy baryon $\mathcal{B}_{QQ}$, if the heavy diquark acts as a point-like constitute, its mass is of the form

$$m_{\mathcal{B}_{QQ}} = 2m_Q + \bar{\Lambda}_{\mathcal{B}_{QQ}} + \frac{\mu_\pi^2}{m_Q} - \frac{\mu_G^2}{m_Q} + \mathcal{O}\left(\frac{1}{m_Q^2} \right). \qquad (2.12)$$

There are two distinct chromomagnetic fields inside the $\mathcal{B}_{QQ}$: one is the chromomagnetic field produced by the light quark and the other by the heavy quark. For the former (latter), the operator $\sigma \cdot G$ is proportional to $\vec{S}_d \cdot \vec{S}_q$ ($\vec{S}_1 \cdot \vec{S}_2$), where $\vec{S}_d = \vec{S}_1 + \vec{S}_2$ ($\vec{S}_q$) is the spin operator of the diquark (light quark), and $\vec{S}_i$ $(i = 1, 2)$ is the spin of the constituent quark inside the diquark. The parameter d_H is given by[2]

$$\begin{aligned} d_H^{dq} &= -4 \langle \mathcal{B}_{QQ} | \vec{S}_d \cdot \vec{S}_q | \mathcal{B}_{QQ} \rangle \\ &= -2[S_{\rm tot}(S_{\rm tot}+1) - S_d(S_d+1) - S_q(S_q+1)], \\ d_H^{QQ} &= -4 \langle \mathcal{B}_{QQ} | \vec{S}_1 \cdot \vec{S}_2 | \mathcal{B}_{QQ} \rangle \\ &= -2[S_d(S_d+1) - S_1(S_1+1) - S_2(S_2+1)]. \end{aligned} \qquad (2.13)$$

Therefore, $d_H^{dq} = 4$, $d_H^{QQ} = -1$ for the spin-$\frac{1}{2}$ doubly heavy baryon $\mathcal{B}_{QQ}$ and $d_H^{dq} = -2$, $d_H^{QQ} = -1$ for the spin-$\frac{3}{2}$ doubly heavy baryon $\mathcal{B}_{QQ}^*$. It follows from Eq. (2.12) that λ_2^{dq} can be expressed in terms of the hyperfine mass splitting

$$\lambda_2^{dq}(\mathcal{B}_{QQ}) = \frac{1}{6} (m_{\mathcal{B}_{QQ}^*} - m_{\mathcal{B}_{QQ}}) m_Q, \qquad (2.14)$$

and hence,

$$\mu_G^2(\mathcal{B}_{QQ}) = \frac{2}{3} (m_{\mathcal{B}_{QQ}^*} - m_{\mathcal{B}_{QQ}}) m_Q - \lambda_2^{QQ}. \qquad (2.15)$$

To evaluate the parameter λ_2^{QQ}, let us consider a simple quark model of De Rújula *et al.* [14]

$$\begin{aligned} M_{\rm baryon} &= M_0 + \cdots + \frac{16}{9} \pi \alpha_s \sum_{i>j} \frac{\vec{S}_i \cdot \vec{S}_j}{m_i m_j} |\psi(0)|^2, \\ M_{\rm meson} &= M_0 + \cdots + \frac{32}{9} \pi \alpha_s \frac{\vec{S}_1 \cdot \vec{S}_2}{m_1 m_2} |\psi(0)|^2. \end{aligned} \qquad (2.16)$$

It is well known that the fine structure constant is $-\frac{4}{3}\alpha_s$ for $\bar{q}q$ pairs in a meson and $-\frac{2}{3}\alpha_s$ for qq pairs in a baryon [14]. This is because the $\bar{q}q$ pair in a meson must be a color-singlet, while the qq pair in a baryon is in color antitriplet state. The mass of the doubly heavy baryon $\mathcal{B}_{QQ}$ is given by

$$\begin{aligned} m_{\mathcal{B}_{QQ}} = 2m_Q + \cdots + \frac{16}{9} \pi \alpha_s \bigg(& \frac{\vec{S}_d \cdot \vec{S}_q}{m_Q m_q} |\psi^{dq}(0)|^2 \\ & + \frac{\vec{S}_1 \cdot \vec{S}_2}{m_Q^2} |\psi^{QQ}(0)|^2 \bigg) + \mathcal{O}\left(\frac{1}{m_Q^2} \right), \end{aligned} \qquad (2.17)$$

where $\psi^{dq}(0)$ is the light quark wave function at the origin of the QQ diquark and $\psi^{QQ}(0)$ is the diquark wave function at the origin. For the doubly charmed baryons we have

$$\begin{aligned} m_{\Xi_{cc}} &= 2m_c + \cdots + \frac{16}{9} \pi \alpha_s \bigg(-\frac{1}{m_c m_q} |\psi^{dq}(0)|^2 \\ &\quad + \frac{1}{4m_c^2} |\psi^{cc}(0)|^2 \bigg) + \mathcal{O}\left(\frac{1}{m_c^2} \right), \\ m_{\Xi_{cc}^*} &= 2m_c + \cdots + \frac{16}{9} \pi \alpha_s \bigg(\frac{1}{2m_c m_q} |\psi^{dq}(0)|^2 \\ &\quad + \frac{1}{4m_c^2} |\psi^{cc}(0)|^2 \bigg) + \mathcal{O}\left(\frac{1}{m_c^2} \right). \end{aligned} \qquad (2.18)$$

The term proportional to $|\psi^{dq}(0)|^2$ can be expressed in terms of the hyperfine mass splitting of Ξ_{cc}. Hence, we obtain

[2] The coefficients of $\vec{S}_d \cdot \vec{S}_q$ and $\vec{S}_1 \cdot \vec{S}_2$ can be arbitrarily chosen. The μ_G^2 term is independent of the choice of d_H.

$$\mu_G^2(\Xi_{cc}) = \frac{2}{3}(m_{\Xi_{cc}^*} - m_{\Xi_{cc}})m_c - \frac{4}{9}\pi\alpha_s \frac{|\psi^{cc}(0)|^2}{m_c} + \mathcal{O}\left(\frac{1}{m_c}\right). \tag{2.19}$$

Hence, $\lambda_2^{cc}(\Xi_{cc}) = (1/9)g_s^2|\psi^{cc}(0)|^2/m_c$.

However, the above expression of μ_G^2 is not the end of story. It has been known that HQET is not the appropriate effective field theory for hadrons with more than one heavy quark. HQET is formulated as an expansion in $\Lambda_{\rm QCD}/m_Q$. For a singly heavy hadron, the heavy quark kinetic energy is neglected as it occurs as a small $1/m_Q$ correction. For a bound state containing two or more heavy quarks, the heavy quark kinetic energy is very important and cannot be treated as a perturbation. The appropriate theory for dealing such a system is non-relativistic QCD (NRQCD),[3] in which one has

$$\bar{Q}g_s\sigma\cdot GQ = -2\psi_Q^\dagger g_s\vec{\sigma}\cdot\vec{B}\psi_Q - \frac{1}{m_Q}\psi_Q^\dagger g_s\vec{D}\cdot\vec{E}\psi_Q + \cdots \tag{2.20}$$

in terms of the two-spinor ψ_Q. According to the counting rule, the Darwin term for the interaction with the chromoelectric field is of the same order of magnitude as the chromomagnetic term [16]. Hence, we get an additional contribution to μ_G^2

$$\mu_G^2 = \frac{2}{3}(m_{\Xi_{cc}^*} - m_{\Xi_{cc}})m_c - \frac{1}{9}g_s^2\frac{|\psi^{cc}(0)|^2}{m_c} - \frac{1}{6}g_s^2\frac{|\psi^{cc}(0)|^2}{m_c}. \tag{2.21}$$

The last term can be obtained by using the equation of motion for the chromoelectric field. Note that our result is different from the original expression[4]

$$\mu_G^2 = -\frac{2}{3}(m_{\Xi_{cc}^*} - m_{\Xi_{cc}})m_c - \frac{2}{9}g_s^2\frac{|\psi^{cc}(0)|^2}{m_c} - \frac{1}{3}g_s^2\frac{|\psi^{cc}(0)|^2}{m_c} \tag{2.22}$$

obtained in [3] in the sign of the first term and in the magnitude of $|\psi^{cc}(0)|^2$ terms. Therefore,

$$\frac{\langle\Xi_{cc}|\bar{c}c|\Xi_{cc}\rangle}{2m_{\Xi_{cc}}} = 1 - \frac{1}{2}v_c^2 + \frac{1}{3}\frac{m_{\Xi_{cc}^*} - m_{\Xi_{cc}}}{m_c} - \frac{2}{9}\pi\alpha_s\frac{|\psi^{cc}(0)|^2}{m_c^3} - \frac{1}{3}\pi\alpha_s\frac{|\psi^{cc}(0)|^2}{m_c^3} \tag{2.23}$$

Since the hyperfine mass splitting of D mesons is given by

$$m_{D^*} - m_D = \frac{32}{9}\alpha_s\pi\frac{|\psi^{c\bar{q}}(0)|^2}{m_c m_q}, \tag{2.24}$$

we are led to the relation

$$m_{\Xi_{cc}^*} - m_{\Xi_{cc}} = \frac{3}{4}(m_{D^*} - m_D)\frac{|\psi^{dq}_{\Xi_{cc}}(0)|^2}{|\psi^{c\bar{q}}_{D_q}(0)|^2}. \tag{2.25}$$

In the heavy quark limit, the doubly charmed baryon wave function $\psi^{dq}_{\Xi_{cc}}(0)$ is expected to be the same as the meson wave function $\psi^{c\bar{q}}_{D_q}(0)$ if the diquark behaves as a point-like particle,[5]

$$\psi^{dq}_{\Xi_{cc}}(0) \approx \psi^{c\bar{q}}_{D_q}(0). \tag{2.26}$$

It follows the well-known mass relation

$$m_{\Xi_{cc}^*} - m_{\Xi_{cc}} = \frac{3}{4}(m_{D^*} - m_D), \tag{2.27}$$

which has been derived in various contents, such as HQET [18],[6] pNRQCD (potential NRQCD) [19,20] and the quark model [21,22].

The nonleptonic and semiletponic decay rates of the heavy quark c of the $\mathcal{B}_{cc}$ are given by

$$\Gamma^{\rm dec}(\mathcal{B}_{cc}) = 2\frac{G_F^2 m_c^5}{192\pi^3}\xi\left\{c_{3,c}^{\rm NL}\left[1 - \frac{\mu_\pi^2}{2m_c^2} + \frac{\mu_G^2}{2m_c^2}\right] + 2c_{5,c}^{\rm NL}\frac{\mu_G^2}{m_c^2}\right\} \tag{2.28}$$

and

$$\Gamma^{\rm SL}(\mathcal{B}_{cc}) = 2\frac{G_F^2 m_c^5}{192\pi^3}\xi\left\{c_{3,c}^{\rm SL}\left[1 - \frac{\mu_\pi^2}{2m_c^2} + \frac{\mu_G^2}{2m_c^2}\right] + 2c_{5,c}^{\rm SL}\frac{\mu_G^2}{m_c^2}\right\}, \tag{2.29}$$

where the expressions of the coefficients $c_{3,c}$ and $c_{5,c}$ can be found, e.g., in [8].

[3] However, it was pointed out very recently in [15] that in the limit $m_Q > m_Q v_Q > m_Q v_Q^2 \gg \Lambda_{\rm QCD}$, such a system can be described by a version of HQET with a diquark degree of freedom.

[4] Guberina *et al.* [5] obtained a similar expression except for the magnitude of $|\psi^{cc}(0)|^2$ terms

$$\mu_G^2 = \frac{2}{3}(m_{\Xi_{cc}^*} - m_{\Xi_{cc}})m_c - \frac{2}{9}g_s^2\frac{|\psi^{cc}(0)|^2}{m_c^*} - \frac{1}{3}g_s^2\frac{|\psi^{cc}(0)|^2}{m_c}.$$

[5] In [5] and in [17], the authors argued that $|\psi^{dq}(0)|^2 = \frac{2}{3}|\psi^{c\bar{q}}(0)|^2$ due to different spin content of doubly charmed baryons. However, this will not lead to the approximate mass relation given by Eq. (2.27).

[6] A factor of 2 was missed in the original mass relation derived in [18].

B. Dimension-six operators

Defining

$$\mathcal{T}_6 = \frac{G_F^2 m_Q^2}{192\pi^3} \xi c_{6,Q}^{\rm NL} T_6, \tag{2.30}$$

the dimension-six four-quark operators in Eq. (2.3) for spectator effects in inclusive decays of doubly charmed baryons denoted by $\mathcal{B}_{cc}$ are given by (only Cabibbo-allowed decays with $\xi = |V_{cs}V_{ud}|^2$ being listed here) [23–25]

$$\begin{aligned}
\mathcal{T}_{6,\rm ann}^{\mathcal{B}_{cc},d} &= \frac{G_F^2 m_c^2}{2\pi}\xi(1-x)^2\{(c_1^2+c_2^2)(\bar{c}c)(\bar{d}d) + 2c_1c_2(\bar{c}d)(\bar{d}c)\},\\
\mathcal{T}_{6,\rm int-}^{\mathcal{B}_{cc},u} &= -\frac{G_F^2 m_c^2}{6\pi}\xi(1-x)^2\left\{c_1^2\left[\left(1+\frac{x}{2}\right)(\bar{c}c)(\bar{u}u) - (1+2x)\bar{c}^\alpha(1-\gamma_5)u^\beta\bar{u}^\beta(1+\gamma_5)c^\alpha\right]\right.\\
&\quad \left. + (2c_1c_2 + N_c c_2^2)\left[\left(1+\frac{x}{2}\right)(\bar{c}u)(\bar{u}c) - (1+2x)\bar{c}(1-\gamma_5)u\bar{u}(1+\gamma_5)c\right]\right\},\\
\mathcal{T}_{6,\rm int+}^{\mathcal{B}_{cc},s} &= -\frac{G_F^2 m_c^2}{6\pi}\xi\{c_2^2[(\bar{c}c)(\bar{s}s) - \bar{c}^\alpha(1-\gamma_5)s^\beta\bar{s}^\beta(1+\gamma_5)c^\alpha]\\
&\quad + (2c_1c_2 + N_c c_1^2)[(\bar{c}s)(\bar{s}c) - \bar{c}(1-\gamma_5)s\bar{s}(1+\gamma_5)c]\},
\end{aligned} \tag{2.31}$$

where $(\bar{q}_1 q_2) \equiv \bar{q}_1\gamma_\mu(1-\gamma_5)q_2$, and α, β are color indices and $x = m_s^2/m_c^2$.

Spectator effects in the weak decays of the doubly charmed baryons Ξ_{cc}^{++}, Ξ_{cc}^{+} and Ω_{cc}^{+} are depicted in Fig. 1. The first term $\mathcal{T}_{6,\rm ann}^{\mathcal{B}_{cc},d}$ in (2.31) corresponds to a W-exchange contribution which appears in Ξ_{cc}^{+} decays (Cabibbo-suppressed $\mathcal{T}_{6,\rm ann}^{\mathcal{B}_{cc},s}$ term appearing in Ω_{cc}^{+} decays). The second term $\mathcal{T}_{6,\rm int-}^{\mathcal{B}_{cc},u}$ arises from the destructive Pauli interference of the u quark produced in the c quark decay with the u quark in the wave function of the doubly charm baryon $\mathcal{B}_{cc}$, namely Ξ_{cc}^{++}

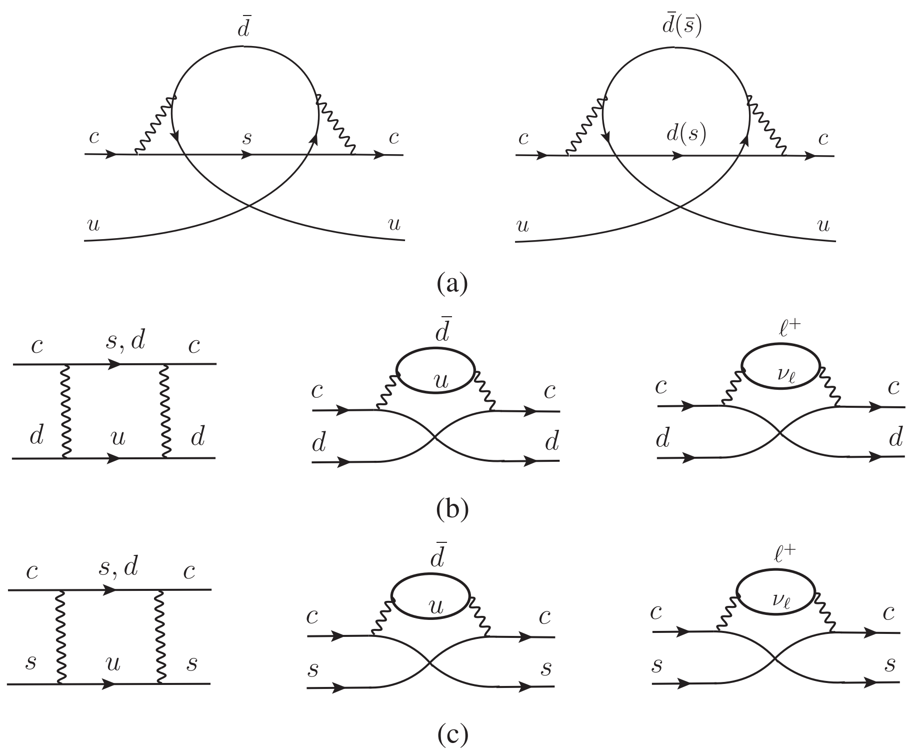

FIG. 1. Spectator effects in doubly charmed baryon decays: (a) destructive Pauli interference in Ξ_{cc}^{++} decay, (b) W-exchange and constructive Pauli interference in Ξ_{cc}^{+} decay, and (c) W-exchange and constructive Pauli interference in Ω_{cc}^{+} decay.

(Fig. 1(a)). The last term $\mathcal{T}^{\mathcal{B}_{cc},s}_{6,\mathrm{int}+}$ is due to the constructive interference of the s quark and hence it occurs only in charmed baryon decays (Fig. 1(c)).

For inclusive semileptonic decays, apart from the heavy quark decay contribution there is an additional spectator effect in charmed-baryon semileptonic decay originating from the Pauli interference of the s or d quark [26]; that is, the s (d) quark produced in $c \to s\ell^+\nu_\ell$ ($c \to d\ell^+\nu_\ell$) has an interference with the s (d) quark in the wave function of the charmed baryon (see Fig. 1). It is now ready to deduce this term from $\mathcal{T}^{q_3}_{6,\mathrm{int}+}$ in Eq. (2.31) by putting $c_1 = 1$, $c_2 = 0$, $N_c = 1$:

$$\mathcal{T}^{\mathrm{SL}}_{6,\mathrm{int}} = -\frac{G_F^2 m_c^2}{6\pi}(|V_{cs}V_{ud}|^2[(\bar{c}s)(\bar{s}c) - \bar{c}(1-\gamma_5)s\bar{s}(1+\gamma_5)c] + |V_{cd}V_{ud}|^2[(\bar{c}d)(\bar{d}c) - \bar{c}(1-\gamma_5)d\bar{d}(1+\gamma_5)c]). \tag{2.32}$$

Before proceeding, we would like to clarify how the heavy quark expansion and approximation are consistent with the claimed accuracy. For example, the hadronic matrix element of the dimension-three operator $\bar{Q}Q$, Eq. (2.6), is in itself an approximation valid up to corrections of order $1/m_Q^3$. This is because the chromomagnetic operator μ_G^2 given in Eq. (2.21), for instance, is valid up to $1/m_Q$ corrections stemming from the expansion of Eq. (2.12) truncated at order $1/m_Q$. Hence, to the order of $1/m_Q^3$ expansion in Eq. (2.3), one may wonder if it is necessary to take into account the higher order corrections such as $c_{3,Q}\mathcal{O}(1/m_Q^3) + c_{5,Q}\mathcal{O}(1/m_Q^3)$ besides the dimension-six operator $c_{6,Q}T_6/m_Q^3$. It turns out that higher order corrections can be neglected as there is a two-body phase-space enhancement factor of $16\pi^2$ for spectator effects induced by dimension-six four-quark operators T_6 relative to the three-body phase space for heavy quark decay. Indeed, the phase-space enhancement for spectator effects is already taken into account in Eq. (2.31). Likewise, higher order corrections $c_{3,Q}\mathcal{O}(1/m_Q^4) + c_{5,Q}\mathcal{O}(1/m_Q^4)$ should be less important than the dimension-seven operators $c_{7,Q}T_6/m_Q^4$.

C. Dimension-seven operators

To the order of $1/m_Q^4$ in the heavy quark expansion in Eq. (2.3), we need to consider dimension-seven operators. For our purposes, we shall focus on the $1/m_Q$ corrections to the spectator effects discussed in the last subsection and neglect the operators with gluon fields. Dimension-seven terms are either the four-quark operators times the spectator quark mass or the four-quark operators with one or two additional derivatives [27,28]. We shall follow [29] to define the following dimension-seven four-quark operators:

$$\begin{aligned}
P_1^q &= \frac{m_q}{m_Q}\bar{Q}(1-\gamma_5)q\bar{q}(1-\gamma_5)Q,\\
P_2^q &= \frac{m_q}{m_Q}\bar{Q}(1+\gamma_5)q\bar{q}(1+\gamma_5)Q,\\
P_3^q &= \frac{1}{m_Q^2}\bar{Q}\overleftarrow{D}_\rho\gamma_\mu(1-\gamma_5)D^\rho q\bar{q}\gamma^\mu(1-\gamma_5)Q,\\
P_4^q &= \frac{1}{m_Q^2}\bar{Q}\overleftarrow{D}_\rho(1-\gamma_5)D^\rho q\bar{q}(1+\gamma_5)Q,\\
P_5^q &= \frac{1}{m_Q}\bar{Q}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma^\mu(1-\gamma_5)(i\not{D})Q,\\
P_6^q &= \frac{1}{m_Q}\bar{Q}(1-\gamma_5)q\bar{q}(1+\gamma_5)(i\not{D})Q,
\end{aligned}$$

and the color-octet operators S_i^q ($i = 1, \ldots, 6$) obtained from P_i^q by inserting t^a in the two currents of the respective color singlet operators. In order to evaluate the baryon matrix elements, it is more convenient to express dimension-seven operators in terms of P_i^q and $\tilde{P}_i^q$ operators, where $\tilde{P}_i$ denotes the color-rearranged operator that follows from the expression of P_i by interchanging the color indices of the q_i and $\bar{q}_j$ Dirac spinors. We shall see below that the hadronic matrix elements of dimension-seven operators are suppressed relative to that of dimension-six ones by order m_q/m_c.

Using the relation

$$S_i = -\frac{1}{2N_c}P_i + \frac{1}{2}\tilde{P}_i, \tag{2.33}$$

we obtain [8]

$$\begin{aligned}
\mathcal{T}^{\mathcal{B}_{cc},d}_{7,\mathrm{ann}} &= \frac{G_F^2 m_c^2}{2\pi}\xi(1-x)\{2c_1c_2[2(1+x)P_3^d + (1-x)P_5^d] + (c_1^2+c_2^2)[2(1+x)\tilde{P}_3^d + (1-x)\tilde{P}_5^d]\},\\
\mathcal{T}^{\mathcal{B}_{cc},u}_{7,\mathrm{int}} &= \frac{G_F^2 m_c^2}{6\pi}\xi(1-x)\Big\{(2c_1c_2 + N_c c_2^2)\\
&\times\Big[-(1-x)(1+2x)(P_1^u+P_2^u) + 2(1+x+x^2)P_3^u\\
&-12x^2P_4^u - (1-x)\left(1+\frac{x}{2}\right)P_5^u + (1-x)(1+2x)P_6^u\Big]\\
&+c_1^2\Big[-(1-x)(1+2x)(\tilde{P}_1^u+\tilde{P}_2^u) + 2(1+x+x^2)\tilde{P}_3^u\\
&-12x^2\tilde{P}_4^u - (1-x)\left(1+\frac{x}{2}\right)\tilde{P}_5^u + (1-x)(1+2x)\tilde{P}_6^u\Big]\Big\},\\
\mathcal{T}^{\mathcal{B}_{cc},s}_{7,\mathrm{int}} &= \frac{G_F^2 m_c^2}{6\pi}\xi\{(2c_1c_2 + N_c c_1^2)[-P_1^s - P_2^s + 2P_3^s - P_5^s + P_6^s]\\
&+c_2^2[-\tilde{P}_1^s - \tilde{P}_2^s + 2\tilde{P}_3^s - \tilde{P}_5^s + \tilde{P}_6^s]\}.
\end{aligned} \tag{2.34}$$

As for the dimension-seven four-operator for semileptonic decays, it can be obtained from $\mathcal{T}^{\mathcal{B}_{cc},s}_{7,\mathrm{int}}$ by setting $c_1 = 1$, $c_2 = 0$ and $N_c = 1$. Taking into account the lepton mass corrections, it reads [8]

$$\mathcal{T}_{7,\mathrm{int}}^{\mathrm{SL}} = \frac{G_F^2 m_c^2}{6\pi}\xi[-(1-z)^2(1+2z)(P_1^s+P_2^s) + 2(1-z)(1+z+z^2)P_3^s - 12z^2(1-z)P_4^s], \tag{2.35}$$

where $z = (m_\ell/m_c)^2$.

III. LIFETIMES OF DOUBLY CHARMED BARYONS

The inclusive nonleptonic rates of doubly charmed baryons in the valence quark approximation and in the limit $m_s/m_c = 0$ can be expressed approximately as

$$\Gamma_{\mathrm{NL}}(\Xi_{cc}^{++}) = \Gamma^{\mathrm{dec}} + \Gamma_-^{\mathrm{int}},$$
$$\Gamma_{\mathrm{NL}}(\Xi_{cc}^{+}) = \Gamma^{\mathrm{dec}} + \cos\theta_C^2\Gamma^{\mathrm{ann}} + \sin\theta_C^2\Gamma_+^{\mathrm{int}},$$
$$\Gamma_{\mathrm{NL}}(\Omega_{cc}^{+}) = \Gamma^{\mathrm{dec}} + \sin\theta_C^2\Gamma^{\mathrm{ann}} + \cos\theta_C^2\Gamma_+^{\mathrm{int}}. \tag{3.1}$$

Because Γ_+^{int} is positive and Γ_-^{int} is negative, it is obvious that Ξ_{cc}^{++} is longest-lived, whereas Ξ_{cc}^{+} (Ω_{cc}^{+}) is the shortest-lived if $\Gamma_+^{\mathrm{int}} > \Gamma^{\mathrm{ann}}$ ($\Gamma_+^{\mathrm{int}} < \Gamma^{\mathrm{ann}}$). In this section, we shall begin with the evaluation of the doubly charmed baryon matrix elements of dimension-six and -seven operators and then proceed to compute the spectator effects to see the relative weight between Γ_+^{int} and Γ^{ann}.

A. Baryon matrix elements

The spectator effects in inclusive heavy bottom baryon decays arising from dimension-six and dimension-seven operators are given by Eqs. (2.31), (2.32), (2.34), and (2.35), respectively. We shall rely on the quark model to evaluate the baryon matrix elements of four-quark operators. The cc diquark of the doubly charmed baryon is of the axial-vector type with spin 1. Hence, the $\mathcal{B}_{cc}$ matrix elements of dimension-six four-quark operators are similar to that of the sextet singly charmed baryon Ω_c^0. Following [8], we write down the relevant baryon matrix elements of dimension-six operators

$$\langle\mathcal{B}_{cc}|(\bar{c}q)(\bar{q}c)|\mathcal{B}_{cc}\rangle = -f_{D_q}^2 m_{D_q} m_{\mathcal{B}_{cc}} r_{\mathcal{B}_{cc}},$$
$$\langle\mathcal{B}_{cc}|(\bar{c}c)(\bar{q}q)|\mathcal{B}_{cc}\rangle = f_{D_q}^2 m_{D_q} m_{\mathcal{B}_{cc}} r_{\mathcal{B}_{cc}}\tilde{B},$$
$$\langle\mathcal{B}_{cc}|\bar{c}(1-\gamma_5)q\bar{q}(1+\gamma_5)c|\mathcal{B}_{cc}\rangle = -\frac{1}{6}f_{D_q}^2 m_{D_q} m_{\mathcal{B}_{cc}} r_{\mathcal{B}_{cc}},$$
$$\langle\mathcal{B}_{cc}|\bar{c}^\alpha(1-\gamma_5)q^\beta\bar{q}^\beta(1+\gamma_5)c^\alpha|\mathcal{B}_{cc}\rangle = \frac{1}{6}f_{D_q}^2 m_{D_q} m_{\mathcal{B}_{cc}} r_{\mathcal{B}_{cc}}\tilde{B}, \tag{3.2}$$

where f_{D_q} and m_{D_q} are the decay constant and the mass of the heavy meson D_q, respectively, and the wave function ratio $r_{\mathcal{B}_{cc}}$ is defined by

$$r_{\Xi_{cc}} \equiv \left|\frac{\psi_{\Xi_{cc}}^{dq}(0)}{\psi_D^{c\bar{q}}(0)}\right|^2 = \frac{4}{3}\frac{m_{\Xi_{cc}^*} - m_{\Xi_{cc}}}{m_{D^*} - m_D},$$
$$r_{\Omega_{cc}} \equiv \left|\frac{\psi_{\Omega_{cc}}^{ds}(0)}{\psi_{D_s}^{c\bar{s}}(0)}\right|^2 = \frac{4}{3}\frac{m_{\Omega_{cc}^*} - m_{\Omega_{cc}}}{m_{D_s^*} - m_{D_s}}. \tag{3.3}$$

According to Eq. (2.26), we should have $r_{\Xi_{cc}} = 1$.[7] The parameter $\tilde{B}$ is defined by

$$\langle\mathcal{B}_{cc}|(\bar{c}c)(\bar{q}q)|\mathcal{B}_{cc}\rangle = -\tilde{B}\langle\mathcal{B}_{cc}|(\bar{c}q)(\bar{q}c)|\mathcal{B}_{cc}\rangle. \tag{3.4}$$

Since the color wavefunction for a baryon is totally antisymmetric, the matrix element of $(\bar{c}c)(\bar{q}q)$ is the same as that of $(\bar{c}q)(\bar{q}c)$ except for a sign difference. That is, $\tilde{B} = 1$ under the valence-quark approximation.

Likewise, the $\mathcal{B}_{cc}$ matrix elements of dimension-seven operators are similar to that of the sextet singly charmed baryon Ω_c^0 (see Eq. (4.14) of [8])

$$\langle\mathcal{B}_{cc}|P_1^q|\mathcal{B}_{cc}\rangle = \langle\mathcal{B}_{cc}|P_2^q|\mathcal{B}_{cc}\rangle = \frac{1}{8}f_{D_q}^2 m_{D_q} m_{\mathcal{B}_{cc}} r_{\mathcal{B}_{cc}}\left(\frac{m_{\mathcal{B}_{cc}}^2 - m_{\{cc\}}^2}{m_c^2}\right)\eta_{1,2}^q,$$
$$\langle\mathcal{B}_{cc}|P_3^q|\mathcal{B}_{cc}\rangle = 6\langle\mathcal{B}_{cc}|P_4^q|\mathcal{B}_{cc}\rangle = -\frac{1}{4}f_{D_q}^2 m_{D_q} m_{\mathcal{B}_{cc}} r_{\mathcal{B}_{cc}}\left(\frac{m_{\mathcal{B}_{cc}}^2 - m_{\{cc\}}^2}{m_c^2}\right)\eta_{3,4}^q, \tag{3.5}$$

where the parameters η_i^q are expected to be of order unity, and $m_{\{cc\}}$ is the mass of the cc diquark. We take $m_{\{cc\}}$ to be 3226 MeV obtained from the relativistic quark model [22]. Note that the term

$$\frac{m_{\mathcal{B}_{cc}}^2 - m_{\{cc\}}^2}{m_c^2} \approx 4\frac{p_c \cdot p_q}{m_c^2} \tag{3.6}$$

is of order m_q/m_c. Therefore, the matrix elements of dimension-seven operators are suppressed by a factor of m_q/m_c relative to that of dimension-six ones. For the matrix elements of the operators $\tilde{P}_i^q$, we introduce a parameter $\tilde{\beta}_i^q$ in analog to Eq. (3.4)

$$\langle\mathcal{B}_{cc}|\tilde{P}_i^q|\mathcal{B}_{cc}\rangle = -\tilde{\beta}_i^q\langle\mathcal{B}_{cc}|P_i^q|\mathcal{B}_{cc}\rangle, \tag{3.7}$$

so that $\tilde{\beta}_i^q = 1$ under the valence quark approximation.

For the spectator effects in doubly charmed baryon decays,

[7] Using the masses of doubly charmed baryons calculated in the relativistic quark model [22], we find numerically $r_{\Xi_{cc}} = 0.99$ and $r_{\Omega_{cc}} = 0.87$.

$$\Gamma^{\rm spec}(\mathcal{B}_{cc}) = \frac{\langle \mathcal{B}_{cc}|\mathcal{T}_6+\mathcal{T}_7|\mathcal{B}_{cc}\rangle}{2m_{\mathcal{B}_{cc}}}, \tag{3.8}$$

we apply Eqs. (3.2) and (3.5) to evaluate the matrix elements of the dimension-six and -seven operators. The results are

$$\begin{aligned}
\Gamma^{\rm ann}(\Omega_{cc}^+) &= 3\frac{G_F^2m_c^2}{\pi}|V_{cs}V_{us}|^2 r_{\Omega_{cc}}|\psi_{c\bar s}^{D_s}(0)|^2\left((1-x)^2+\left|\frac{V_{cd}}{V_{cs}}\right|^2\right)\Bigg\{(\tilde B(c_1^2+c_2^2)-2c_1c_2) \\
&\quad +\frac{1}{2}(\tilde\beta(c_1^2+c_2^2)-2c_1c_2)\eta\left(\frac{m_{\Omega_{cc}}^2-m_{\{cc\}}^2}{m_c^2}\right)\Bigg\}, \\
\Gamma^{\rm ann}(\Xi_{cc}^+) &= 3\frac{G_F^2m_c^2}{\pi}|V_{cs}V_{ud}|^2 r_{\Xi_{cc}}|\psi_{c\bar d}^{D}(0)|^2\left((1-x)^2+\left|\frac{V_{cd}}{V_{cs}}\right|^2\right)\Bigg\{(\tilde B(c_1^2+c_2^2)-2c_1c_2) \\
&\quad +\frac{1}{2}(\tilde\beta(c_1^2+c_2^2)-2c_1c_2)\eta\left(\frac{m_{\Xi_{cc}}^2-m_{\{cc\}}^2}{m_c^2}\right)\Bigg\}, \\
\Gamma_+^{\rm int}(\Omega_{cc}^+) &= \frac{G_F^2m_c^2}{6\pi}|V_{cs}V_{ud}|^2 r_{\Omega_{cc}}|\psi_{c\bar s}^{D_s}(0)|^2\Bigg\{(2c_1c_2+N_cc_1^2-\tilde Bc_2^2)\left(5+\left|\frac{V_{us}}{V_{ud}}\right|^2(1-x)^2(5+x)\right) \\
&\quad -\frac{9}{2}(2c_1c_2+N_cc_1^2-\tilde\beta c_2^2)\eta\left(\frac{m_{\Omega_{cc}}^2-m_{\{cc\}}^2}{m_c^2}\right)\Bigg\}, \\
\Gamma_+^{\rm int}(\Xi_{cc}^+) &= \frac{G_F^2m_c^2}{6\pi}|V_{cd}V_{ud}|^2 r_{\Xi_{cc}}|\psi_{c\bar q}^{D}(0)|^2\Bigg\{(2c_1c_2+N_cc_1^2-\tilde Bc_2^2)\left(5+\left|\frac{V_{us}}{V_{ud}}\right|^2(1-x)^2(5+x)\right) \\
&\quad -\frac{9}{2}(2c_1c_2+N_cc_1^2-\tilde\beta c_2^2)\left(1+\left|\frac{V_{us}}{V_{ud}}\right|^2(1-x)^2(1+x)\right)\eta\left(\frac{m_{\Xi_{cc}}^2-m_{\{cc\}}^2}{m_c^2}\right)\Bigg\}, \\
\Gamma_-^{\rm int}(\Xi_{cc}^{++}) &= -\frac{G_F^2m_c^2}{6\pi}|V_{cs}V_{ud}|^2 r_{\Xi_{cc}}|\psi_{c\bar q}^{D}(0)|^2\Bigg\{(\tilde Bc_1^2-2c_1c_2-N_cc_2^2)\left((1-x)^2(5+x)+\left|\frac{V_{cd}}{V_{cs}}\right|^2\right. \\
&\quad \left.+\left|\frac{V_{us}}{V_{ud}}\right|^2\sqrt{1-4x}\right)-\frac{9}{2}(\tilde\beta c_1^2-2c_1c_2-N_cc_2^2)(1-x)\left(1+x-\frac{2}{3}x^2\right)\eta\left(\frac{m_{\Xi_{cc}}^2-m_{\{cc\}}^2}{m_c^2}\right)\Bigg\},
\end{aligned} \tag{3.9}$$

and

$$\begin{aligned}
\Gamma_{\rm int}^{\rm SL}(\Omega_{cc}^+) &= \frac{G_F^2m_c^2}{6\pi}|V_{cs}|^2 r_{\Omega_{cc}}|\psi_{c\bar s}^{D_s}(0)|^2\left[5-\frac{9}{2}\left(1-\frac{5}{6}z^2+\frac{1}{3}z^3\right)\left(\frac{m_{\Omega_{cc}}^2-m_{\{cc\}}^2}{m_c^2}\right)\right], \\
\Gamma_{\rm int}^{\rm SL}(\Xi_{cc}^+) &= \frac{G_F^2m_c^2}{6\pi}|V_{cd}|^2 r_{\Xi_{cc}}|\psi_{c\bar d}^{D}(0)|^2\left[5-\frac{9}{2}\left(1-\frac{5}{6}z^2+\frac{1}{3}z^3\right)\left(\frac{m_{\Xi_{cc}}^2-m_{\{cc\}}^2}{m_c^2}\right)\right].
\end{aligned} \tag{3.10}$$

Except for the weak annihilation term, the expression of Pauli interference will be very lengthy if the hadronic parameters η_i^q and $\tilde\beta_i^q$ are all treated to be different from each other. Since in realistic calculations we will set $\tilde\beta_i^q(\mu_h)=1$ under valence quark approximation and put η_i^q to unity, we shall assume for simplicity that $\eta_i^q=\eta$ and $\tilde\beta_i^q=\tilde\beta$.

As far as the dimension-six spectator effects are concerned, we now compare our results Eqs. (3.9) and (3.10) with Eqs. (13) and (8) of [5]. Since we are working at the $\mu=m_Q$ scale, we need to set the parameter κ appearing in [5] to be unity. Noting that $r_{\mathcal{B}_{cc}}|\psi_D^{c\bar q}(0)|^2=|\psi_{\mathcal{B}_{cc}}^{dq}(0)|^2$ in our case, we see that $\Gamma_+^{\rm int}$ and $\Gamma_-^{\rm int}$ obtained by Guberina, Melić and H. Štefančić (GMS) are larger than ours by a factor of $3/2$, whereas their $\Gamma^{\rm ann}$ ($\Gamma^{\rm SL}$) is smaller than ours by a factor of $6/5$ (2). Because the wave function of the doubly charmed baryon is related to that of the charmed meson through the relation $|\psi^{dq}(0)|^2=\frac{2}{3}|\psi^{c\bar q}(0)|^2$ by GMS, it turns out that while we agree on the $\Gamma_+^{\rm int}$ and $\Gamma_-^{\rm int}$ in terms of $|\psi^{c\bar q}(0)|^2$, the expressions of $\Gamma^{\rm ann}$ and $\Gamma_{\rm int}^{\rm SL}$ by GMS are smaller than ours by a factor of $9/5$ and 3, respectively.

B. Numerical results

To compute the decay widths of doubly charmed baryons, we have to specify the values of $\tilde B$ and $r_{\mathcal{B}_{cc}}$.

TABLE II. Various contributions to the decay rates (in units of 10^{-12} GeV) of doubly charmed baryons to order $1/m_c^3$ with the hadronic scale $\mu_{\rm had} = 0.90$ GeV.

	$\Gamma^{\rm dec}$	$\Gamma^{\rm ann}$	$\Gamma^{\rm int}_-$	$\Gamma^{\rm int}_+$	$\Gamma^{\rm semi}$	$\Gamma^{\rm tot}$	$\tau(10^{-13}\ {\rm s})$	$\tau_{\rm expt}(10^{-13}\ {\rm s})$
Ξ_{cc}^{++}	2.198		−1.383		0.450	1.265	5.20	$2.56^{+0.28}_{-0.26}$
Ξ_{cc}^{+}	2.198	8.628		0.123	0.525	11.475	0.57	
Ω_{cc}^{+}	2.148	0.611		3.217	2.445	8.421	0.78	

Since $\tilde{B} = 1$ in the valence-quark approximation and since the wavefunction squared ratio r is evaluated using the quark model, it is reasonable to assume that the NQM and the valence-quark approximation are most reliable when the baryon matrix elements are evaluated at a typical hadronic scale $\mu_{\rm had}$. As shown in [30], the parameters $\tilde{B}$ and r renormalized at two different scales are related via the renormalization group equation to be

$$\tilde{B}(\mu) r(\mu) = \tilde{B}(\mu_{\rm had}) r(\mu_{\rm had}),$$
$$\tilde{B}(\mu) = \frac{\tilde{B}(\mu_{\rm had})}{\kappa + \frac{1}{N_c}(\kappa - 1)\tilde{B}(\mu_{\rm had})}, \qquad (3.11)$$

with

$$\kappa = \left(\frac{\alpha_s(\mu_{\rm had})}{\alpha_s(\mu)}\right)^{3N_c/2\beta_0} = \sqrt{\frac{\alpha_s(\mu_{\rm had})}{\alpha_s(\mu)}} \qquad (3.12)$$

and $\beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f$. The parameter κ takes care of the evolution from m_Q to the hadronic scale. We consider the hadronic scale in the range of $\mu_{\rm had} \sim 0.65$–1 GeV. Taking the scale $\mu_{\rm had} = 0.90$ GeV as an illustration, we obtain $\alpha_s(\mu_{\rm had}) = 0.59$, $\tilde{B}(\mu) = 0.75\tilde{B}(\mu_{\rm had}) \simeq 0.75$ and $r(\mu) \simeq 1.33 r(\mu_{\rm had})$. The parameter $\tilde{\beta}$ is treated in a similar way.

For numerical calculations, we use $c_1(\mu) = 1.346$ and $c_2(\mu) = -0.636$ evaluated at the scale $\mu = 1.25$ GeV with $\Lambda^{(4)}_{\overline{\rm MS}} = 325$ MeV [31], $m_{\Xi_{cc}^*} - m_{\Xi_{cc}} = 106$ MeV and $m_{\Omega_{cc}^*} - m_{\Omega_{cc}} = 94$ MeV from [22], the wave function $|\psi^{cc}(0)| = 0.17\ {\rm GeV}^{3/2}$ and the average kinetic energy $T = 0.37$ GeV from [3]. For the decay constants, we use $f_D = 204$ MeV and $f_{D_s} = 250$ MeV. For the charmed quark mass we use $m_c = 1.56$ GeV fixed from the experimental values for D^+ and D^0 semileptonic widths [8].

The results of calculations to order $1/m_c^3$ are exhibited in Table II. The lifetime hierarchy $\tau(\Xi_{cc}^{++}) > \tau(\Omega_{cc}^{+}) > \tau(\Xi_{cc}^{+})$ is understandable. The Ξ_{cc}^{++} baryon is longest-lived owing to the destructive Pauli interference, while Ξ_{cc}^{+} is shortest-lived due to the fact that $\Gamma^{\rm ann}(\Xi_{cc}^{+}) \gg \Gamma^{\rm int}_+(\Omega_{cc}^{+})$. From Eq. (3.9) we see that apart from QCD corrections to Wilson coefficients, $\Gamma^{\rm ann}/\Gamma^{\rm int}_+$ is basically of order 18/5. As for semileptonic decay rates, we have $\Gamma^{\rm SL}(\Omega_{cc}^{+}) \gg \Gamma^{\rm SL}(\Xi_{cc}^{+}) > \Gamma^{\rm SL}(\Xi_{cc}^{++})$ owing to a large Pauli interference effect in the Ω_{cc}^{+} but Cabibbo-suppressed in the Ξ_{cc}^{+}. We have checked the lifetimes of doubly charmed baryons against the hadronic scale $\mu_{\rm had}$. The Ξ_{cc}^{++} lifetime remains nearly constant, $\tau(\Xi_{cc}^{+})$ is increased by 10%, while $\tau(\Omega_{cc}^{+})$ increased by 35% when the hadronic scale varies from 0.65 to 1.0 GeV.

As shown in [8], the heavy quark expansion in $1/m_c$ does not work well for describing the lifetime pattern of singly charmed baryons. Since the charm quark is not heavy enough, it is sensible to consider the subleading $1/m_c$ corrections to spectator effects as depicted in Eq. (3.9). The numerical results are shown in Table III. By comparing Table III with Table II, we see that the lifetimes of Ξ_{cc}^{++} and Ξ_{cc}^{+} become shorter, while $\tau(\Omega_{cc}^{+})$ becomes longer. This is because $\Gamma^{\rm int}_+$ and $\Gamma^{\rm semi}$ for Ω_{cc}^{+} are subject to large cancellation between dimension-six and -seven operators. Such cancellation also occurs in Ξ_{cc}^{+} but not so dramatic as the constructive Pauli interference there is Cabibbo-suppressed. We see from Table III that $\Gamma^{\rm int}_+(\Omega_{cc}^{+})$ even becomes negative. This is because the dimension-seven contribution $\Gamma^{\rm int}_{+,7}(\Omega_{cc}^{+})$ is destructive and its size is so large that it overcomes the dimension-six one and flips the sign. This implies that the subleading corrections are too large to justify the validity of the HQE.

In order to allow a description of the $1/m_c^4$ corrections to $\Gamma(\Omega_{cc}^{+})$ within the realm of perturbation theory, we follow [8] to introduce a parameter α so that $\Gamma^{\rm int}_{+,7}(\Omega_{cc}^{+}, \Xi_{cc}^{+})$ and

TABLE III. Various contributions to the decay rates (in units of 10^{-12} GeV) of doubly charmed baryons to order $1/m_c^4$ with the hadronic scale $\mu_{\rm had} = 0.90$ GeV.

	$\Gamma^{\rm dec}$	$\Gamma^{\rm ann}$	$\Gamma^{\rm int}_-$	$\Gamma^{\rm int}_+$	$\Gamma^{\rm semi}$	$\Gamma^{\rm tot}$	$\tau(10^{-13}\ {\rm s})$	$\tau_{\rm expt}(10^{-13}\ {\rm s})$
Ξ_{cc}^{++}	2.198		−0.437		0.451	2.212	2.98	$2.56^{+0.28}_{-0.26}$
Ξ_{cc}^{+}	2.198	12.260		0.030	0.469	14.958	0.44	
Ω_{cc}^{+}	2.148	0.979		−0.246	0.318	3.200	2.06	

TABLE IV. Various contributions to the decay rates (in units of 10^{-12} GeV) of the Ω_{cc}^+ after including subleading $1/m_c$ corrections to spectator effects. However, the dimension-seven contributions $\Gamma_{+,7}^{\rm int}$ and $\Gamma_7^{\rm SL}$ are multiplied by a factor of $(1-\alpha)$ with α varying from 0 to 1.

α	$\Gamma^{\rm dec}$	$\Gamma^{\rm ann}$	$\Gamma_+^{\rm int}$	$\Gamma^{\rm semi}$	$\Gamma^{\rm tot}$	$\tau(10^{-13}$ s)
0	2.148	0.979	−0.246	0.318	3.200	2.06
0.08	2.148	0.979	0.031	0.489	3.647	1.80
0.30	2.148	0.979	0.792	0.956	4.876	1.35
1	2.148	0.979	3.217	2.445	8.789	0.75

$\Gamma_7^{\rm SL}(\Omega_{cc}^+,\Xi_{cc}^+)$ are multiplied by a factor of $(1-\alpha)$; that is, α describes the degree of suppression. In Table IV, we show the variation of the Ω_{cc}^+ lifetime with α. At $\alpha=0.08$, $\Gamma_+^{\rm int}(\Omega_{cc}^+)$ starts to become positive, where $\tau(\Omega_{cc}^+)=1.80\times10^{-13}$ s. Since we do not know what is the value of α, we can only conjecture that the Ω_{cc}^+ lifetime lies in the range

$$0.75\times10^{-13}\ {\rm s}<\tau(\Omega_{cc}^+)<1.80\times10^{-13}\ {\rm s}. \qquad (3.13)$$

For the Ξ_{cc}^+, its lifetime is rather insensitive to the variation of α as both $\Gamma_{+,7}^{\rm int}(\Xi_{cc}^+)$ and $\Gamma_7^{\rm SL}(\Xi_{cc}^+)$ are Cabibbo-suppressed.

Our prediction of $\tau(\Xi_{cc}^{++})$ is slightly larger than the LHCb measurement given by Eq. (1.1). We learn from [8] that the predicted lifetimes of heavy mesons or baryons are always longer than the measured values. Presumably, this is because we have not yet taken into account all possible QCD corrections fully. Nevertheless, the lifetime ratios should be more trustworthy than the absolute lifetimes themselves. In the present work, we find that the ratio $\tau(\Xi_{cc}^{++})/\tau(\Xi_{cc}^+)$ is ~9.1 to order $1/m_c^3$ and ~6.7 to order $1/m_c^4$.

IV. CONCLUSIONS

In this work, we have analyzed the lifetimes of doubly charmed hadrons within the framework of the heavy quark expansion. It is well known that the lifetime differences stem from spectator effects such as W-exchange and Pauli interference. We rely on the quark model to evaluate the hadronic matrix elements of dimension-six and -seven four-quark operators responsible for spectator effects.

The main results of our analysis are as follows.

(i) The doubly charmed baryon matrix element of the $\sigma\cdot G$ operator receives three distinct contributions: the interaction of the heavy quark with the chromomagnetic field produced from the light quark and from the other heavy quark, and the so-called Darwin term in which the heavy quark interacts with the chromoelectric field. The last term arises because the appropriate theory for dealing hadrons with more than one heavy quark is NRQCD rather than HQET.

(ii) The Ξ_{cc}^{++} baryon is longest-lived in the doubly charmed baryon system owing to the destructive Pauli interference absent in the Ξ_{cc}^+ and Ω_{cc}^+. In the presence of dimension-seven contributions, its lifetime is reduced from $\sim5.2\times10^{-13}$ s to $\sim3.0\times10^{-13}$ s.

(iii) The Ξ_{cc}^+ baryon has the shortest lifetime of order 0.45×10^{-13} s due to a large contribution from the W-exchange box diagram.

(iv) It is difficult to make a precise statement on the lifetime of Ω_{cc}^+. Contrary to Ξ_{cc} baryons, $\tau(\Omega_{cc}^+)$ becomes longer in the presence of dimension-7 effects so that the Pauli interference $\Gamma_+^{\rm int}$ even becomes negative. This means that the subleading corrections are too large to justify the validity of the HQE. Demanding the rate $\Gamma_+^{\rm int}$ to be positive for a sensible HQE, we conjecture that the Ω_c^0 lifetime lies in the range of $(0.75\sim1.80)\times10^{-13}$ s.

(v) The lifetime hierarchy pattern is $\tau(\Xi_{cc}^{++})>\tau(\Omega_{cc}^+)>\tau(\Xi_{cc}^+)$ and the lifetime ratio $\tau(\Xi_{cc}^{++})/\tau(\Xi_{cc}^+)$ is predicted to be of order 6.7.

ACKNOWLEDGMENTS

We would like to thank Robert Shrock for helpful discussions. H. Y. C. wishes to thank the hospitality of the C. N. Yang Institute for Theoretical Physics, Stony Brook University. The research of H. Y. C. and Y. L. S. was supported in part by the Ministry of Science and Technology of R.O.C. under Grant No. 106-2112-M-001-015 and U.S. National Science Foundation Grant No. NSF-PHY-16-1620628, respectively.

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