Study of possible molecular states of $D_s^{(*)}D_s^{(*)}$ and $B_s^{(*)}B_s^{(*)}$

Hong-Wei Ke[1,*] and Yan-Liang Shi[2,†]

[1]*School of Science, Tianjin University, Tianjin 300072, China*
[2]*Cold Spring Harbor Laboratory, Cold Spring Harbor, New York 11724, USA*

(Received 7 March 2022; accepted 4 June 2022; published 15 June 2022)

Recently the LHCb Collaboration reported a new exotic state T_{cc}^+ which is conjectured to be a molecular state of D^0D^{*+} (or $D^{*0}D^+$) theoretically. Belle Collaboration also searched for tetraquark state X_{ccss} in D_sD_s ($D_s^*D_s^*$) final states but no significant signals were observed, which did not rule out the existence of X_{ccss} as a molecular state of D_sD_s ($D_s^*D_s^*$). Inspired by these experimental results on double charmed exotic state, in this paper we study whether the molecular bound states of $D_s^{(*)}D_s^{(*)}$ and $B_s^{(*)}B_s^{(*)}$ can exist with the Bethe-Salpeter (BS) equation approach. We employ heavy meson chiral perturbation theory and one-boson-exchange approximation to calculate the interaction kernels in the BS equations. Our numerical results suggest that two B_s^* mesons perhaps form a 0^+ molecular state. Future experimental search for X_{ccss} and X_{bbss} states in other decay channels may shed light on the structure of double charmed exotic state.

DOI: 10.1103/PhysRevD.105.114019

I. INTRODUCTION

Several months ago the LHCb Collaboration declared a new exotic state T_{cc}^+ from the $D^0D^0\pi^+$ final state, which indicates T_{cc}^+ possesses a $cc\bar{u}\,\bar{d}$ flavor component. Since its mass is very close to the mass threshold of D^0D^{*+} and its width is very narrow [1,2], many authors suggested that T_{cc}^+ could be a loose D^0D^{*+} (D^+D^{*0}) bound state [3–15]. Since 2003, many exotic states [16–22] have been observed, such as $X(3872)$, $X(3940)$, $Y(3940)$, $Z(4430)^{\pm}$, $Z_{cs}(4000)$, $Z_{cs}(4220)$, Z_b, Z_b', $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, but heavy quarks in them are hidden. If T_{cc}^+ is confirmed, it will be the first exotic state with two open heavy quarks.

Given a potential T_{cc}^+ state, naturally one would ask whether the exotic states with $cc\bar{s}\,\bar{s}$ flavor component can also exist. Some theoretical works have explored these exotic states [23–26]. Recently Belle Collaboration searched for tetraquark state X_{ccss} in $D_sD_s(D_s^*D_s^*)$ final states but no significant signals were observed [27]. However, we cannot rule out the existence of X_{ccss} as a D_sD_s ($D_s^*D_s^*$) molecular state from Belle's experiment because a ground molecular state cannot decay to its two components.

*khw020056@tju.edu.cn
†shi@cshl.edu

In a recent paper [28], we study the possible bound states of D^0D^+, D^0D^{*+}, and $D^{*0}D^{*+}$ (B^0B^+, B^0B^{*+}, and $B^{*0}B^{*+}$) systems within the Bethe-Salpeter (BS) framework, where the relativistic corrections are automatically included. In this work we follow the same approach to explore the possible bound state of D_sD_s, $D_sD_s^*$, or $D_s^*D_s^*$ (B_sB_s, $B_sB_s^*$, or $B_s^*B_s^*$) system.

Apart from the bound state of two fermions, the BS equation has also been employed to explore the bound state made of one fermion and one boson [29–31] and the system composed of two bosons [32–39]. In Refs. [32–36] two components in the bound state are one particle and one antiparticle. In Ref. [28] the systems composed of two charmed (or bottomed) hadrons are studied. Following the approach in Ref. [28], we investigate the possible bound state of $D_s^{(*)}D_s^{(*)}$ ($B_s^{(*)}B_s^{(*)}$).

In this work we use the heavy meson chiral perturbation theory [40–45] to describe the interaction between constituent mesons. Then we apply one-boson-exchange approximation of effective interaction to calculate the interaction kernels for the BS equation. For $D_s^{(*)}D_s^{(*)}$ and $B_s^{(*)}B_s^{(*)}$ systems, the exchanged particles are light mesons, such as η and ϕ. We ignore the contribution from σ exchange because some authors indicated that makes a secondary contribution [40]. By calculating the corresponding Feynman diagrams for the effective interaction, one can obtain the analytical form of interaction kernel and deduce the BS equation. With input parameters, the BS equation is solved numerically in momentum space. In the case where we cannot find a solution that satisfies the equation given a reasonable range of parameters, the proposed bound state cannot exist. On the contrary, a solution of the BS equation with reasonable parameters implies that the interaction between two constituents is attractive and large enough, i.e., the corresponding bound state could be formed.

After the Introduction, we deduce the BS equations and the corresponding kernels for the D_sD_s, $D_sD_s^*$, and $D_s^*D_s^*$ (B_sB_s, $B_sB_s^*$, and $B_s^*B_s^*$) systems with defined quantum numbers. Then in Sec. III we present our numerical results along with explicitly displaying all input parameters. Section IV is devoted to a brief summary.

II. THE BETHE-SALPETER FORMALISM

In this work we are only concerned with the ground state where the orbital angular momentum between two constituent mesons is zero (i.e., $l = 0$). For a system made of D_s and D_s^* (or B_s and B_s^*), its J^P is 1^+. For the molecular state that consists of D_s and D_s (B_s and B_s), their J^P may be 0^+. However, for the molecular state that consists of D_s^* and D_s^* (B_s^* and B_s^*), their J^P may be 0^+ or 2^+ rather than 1^+ because the total wave function for the combined system of D_s^* and D_s^* (B_s^* and B_s^*) must be symmetric under group $O(3) \times SU_S(2)$, where $SU_S(2)$ is the spin group.

A. The BS equation of 0^+ that is composed of two pseudoscalars

The BS wave function for the bound state $|S\rangle$ of two pseudoscalar mesons can be defined as follows:

$$\langle 0|T\phi_1(x_1)\phi_2(x_2)|S\rangle = \chi_S(x_1, x_2), \tag{1}$$

where $\phi_1(x_1)$ and $\phi_2(x_2)$ are the field operators of two mesons, respectively.

After some manipulations, we obtain the BS equation in the momentum space

$$\chi_S(p) = \Delta_1 \int \frac{d^4p'}{(2\pi)^4} K_S(p, p')\chi_S(p')\Delta_2, \tag{2}$$

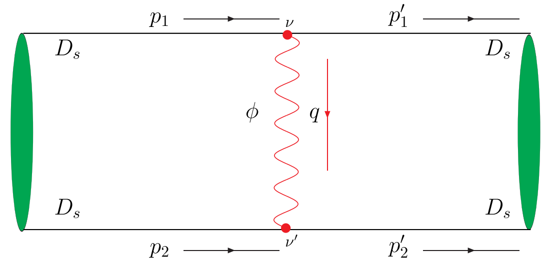


FIG. 1. The leading-order interaction diagram of a bound state composed of two pseudoscalars.

where $\Delta_1 = \frac{i}{p_1^2 - m_1^2}$ and $\Delta_2 = \frac{i}{p_2^2 - m_2^2}$ are the propagators of two pseudoscalar mesons.

The relative momenta and the total momentum of the bound state in the equation are defined as

$$\begin{aligned} &p = \eta_2 p_1 - \eta_1 p_2, \qquad p' = \eta_2 p_1' - \eta_1 p_2', \\ &P = p_1 + p_2 = p_1' + p_2', \end{aligned} \tag{3}$$

where $\eta_i = m_i/(m_1 + m_2)$, P denotes the total momentum of the bound state, and m_i ($i = 1, 2$) is the mass of the ith constituent meson.

Since only $l = 0$ is considered and the total wave function of D_sD_s is symmetric, the J^P of the D_sD_s system is 0^+. According to heavy meson chiral perturbation theory [40–45], the exchanged mesons between the two pseudoscalars are vector mesons, and here we only keep the lightest vector meson ϕ [35,36].

With the Feynman diagrams depicted in Fig. 1 and the effective interactions shown in the Appendix, we obtain the interaction kernel

$$\begin{aligned} K_S(p, p') &= \sqrt{2}K_{S0}(p, p', m_\phi), \\ K_{S0}(p, p', m_V) &= iC_{S0}g_{DDV}^2 \frac{(p_1 + p_1')\cdot(p_2 + p_2') - (p_1 + p_1')\cdot q(p_2 + p_2')\cdot q/m_V^2}{q^2 - m_V^2} F(q)^2, \end{aligned} \tag{4}$$

where $q = p_1 - p_1'$, $C_{S0} = 1$, and m_V stands for the mass of the vector meson. Because two D_s mesons are identical particles, p_1' and p_2' can exchange positions in Fig. 1 an additional $\sqrt{2}$ factor appears in the right side of the first equation (a similar factor also applies to the $D_s^*D_s^*$ systems in Sec. II D). Since the constituent meson is not a point particle, a form factor at each interaction vertex among hadrons must be introduced to reflect the finite-size effects of these hadrons. The form factor is assumed to be in the following form:

$$F(k) = \frac{\Lambda^2 - M_{ex}^2}{\Lambda^2 - k^2}, \tag{5}$$

where Λ is a cutoff parameter, and M_{ex} is the mass of intermediate particle.

Solving Eq. (2) is rather difficult. In general, one needs to use the so-called instantaneous approximation: $p_0' = p_0 = 0$ for $K_0(p, p')$ by which the BS equation can be reduced to

$$\frac{E^2-(E_1+E_2)^2}{(E_1+E_2)/E_1E_2}\psi_S(\mathbf{p})=\frac{i}{2}\int\frac{d^3\mathbf{p}'}{(2\pi)^3}K_S(\mathbf{p},\mathbf{p}')\psi_S(\mathbf{p}'), \quad (6)$$

where $E_i \equiv \sqrt{\mathbf{p}^2+m_i^2}$, $E=P^0$, and the equal-time wave function is defined as $\psi_S(\mathbf{p})=\int dp^0\chi_S(p)$. For exchange of a light vector between the mesons, the kernel is

$$K_S(\mathbf{p},\mathbf{p}')=\sqrt{2}K_{S0}(\mathbf{p},\mathbf{p}',m_\phi), \quad (7)$$

where the expressions of $K_{S0}(\mathbf{p},\mathbf{p}',m_V)$ can be found in Ref. [28].

B. The BS equation of 1^+ that is composed of a pseudoscalar and a vector

The BS wave function for the bound state $|V\rangle$ composed of one pseudoscalar and one vector meson is defined as follows:

$$\langle 0|\mathrm{T}\phi_1(x_1)\phi_2^\mu(x_2)|V\rangle=\chi_V(x_1,x_2)\epsilon^\mu, \quad (8)$$

where ϵ is the polarization vector of the bound state, μ is Lorentz index, and $\phi_1(x_1)$ and $\phi_2^\mu(x_2)$ are the field operators of the pseudoscalar and vector mesons, respectively. The equation for the BS wave function is

$$\chi_V(p)\epsilon^\mu=\Delta_1\int\frac{d^4p'}{(2\pi)^4}K_{V\alpha\beta}(p,p')\chi_V(p')\epsilon^\beta\Delta_{2\mu\alpha}. \quad (9)$$

Here $\Delta_1=\frac{i}{p_1^2-m_1^2}$ and $\Delta_{2\mu\alpha}=\frac{i}{p_2^2-m_2^2}(\frac{p_{2\mu}p_{2\alpha}}{m_2^2}-g_{\mu\alpha})$. We multiply an ϵ_μ^* on both sides and sum over the polarizations, and then we deduce the following equation:

$$\chi_V(p)=\frac{-1}{3(p_1^2-m_1^2)(p_2^2-m_2^2)}\int\frac{d^4p'}{(2\pi)^4}K_{V\alpha\beta}(p,p')\chi_V(p')\left(\frac{p_2^\mu p_2^\alpha}{m_2^2}-g^{\mu\alpha}\right)\left(\frac{P_\mu P^\beta}{M^2}-g_\mu^\beta\right). \quad (10)$$

With the Feynman diagrams depicted in Figs. 2 and 3, we eventually obtain

$$\begin{aligned}
K_{V\alpha\beta}(p,p')&=K_{V1\alpha\beta}(p,p',m_\phi)+K_{V2\alpha\beta}(p,p',m_\phi)+K_{V3\alpha\beta}(p,p',m_\eta),\\
K_{V1\alpha\beta}(p,p',m_V)&=C_{V1}g_{DDV}\Bigg\{g_{D^*D^*V}\left[g_{\alpha\beta}(p_2+p_2')\cdot(p_1+p_1')+g_{\alpha\beta}\frac{q\cdot(p_1+p_1')q\cdot(p_2+p_2')}{m_V^2}\right]\\
&\quad+2g'_{D^*D^*V}[q^\alpha(p_1+p_1')^\beta-q^\beta(p_1+p_1')^\alpha]\Bigg\}\frac{i}{q^2-m_V^2}F(q)^2,\\
K_{V2\alpha\beta}(p,p',m_V)&=C_{V2}\varepsilon^{\mu\nu\beta\tau}(q'_\nu g_{\lambda\mu}-q'_\mu g_{\lambda\nu})(p_2'-p_1)_\tau\varepsilon^{\mu'\nu'\alpha\tau'}(q'_\mu g_{\lambda'\nu}-q'_\nu g_{\lambda'\mu})(p_2-p_1')_{\tau'}\frac{i}{q'^2-m_V^2}(-g^{\lambda\lambda'}+q'^\lambda q'^{\lambda'}/m_V^2)F(q')^2,\\
K_{V3\alpha\beta}(p,p',m_P)&=C_{V3}g_{DD^*P}^2\frac{i}{q^2-m_P^2}q_\alpha q_\beta F(q)^2,
\end{aligned} \quad (11)$$

where $q'=p_1-p_2'$, m_V represents the mass of vector meson (e.g., ϕ), and m_P represents the mass of pseudoscalar meson (e.g., η). The contributions from Fig. 2 are included in $K_{V3\alpha\beta}(p,p',m_\eta)$, and those from Figs. 3(a) and 3(b) are included in $K_{V1\alpha\beta}(p,p',m_\phi)$ and $K_{V2\alpha\beta}(p,p',m_\phi)$, respectively. The coefficients C_{V1}, C_{V2}, and C_{V3} are 1, 1, and $\frac{2}{3}$, respectively.

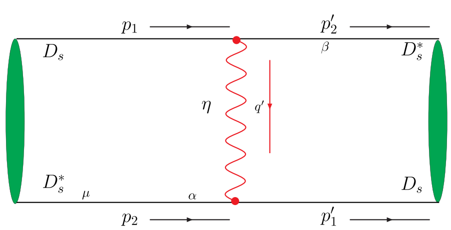


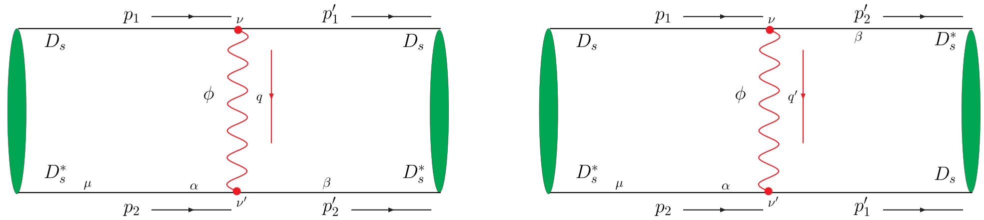
FIG. 2. The leading-order interaction diagram of a bound state composed of a pseudoscalar and a vector by exchanging η.

Defining $K_V(p,p')=K_{V\alpha\beta}(q)(\frac{p_2^\mu p_2^\alpha}{m_2^2}-g^{\mu\alpha})(\frac{P_\mu P^\beta}{M^2}-g_\mu^\beta)$ and setting $p_0=q_0=0$, we derive the BS equation that is similar to Eq. (6) but possesses a different kernel,

$$\frac{E^2-(E_1+E_2)^2}{(E_1+E_2)/E_1E_2}\psi_V(\mathbf{p})=\frac{i}{2}\int\frac{d^3\mathbf{p}'}{(2\pi)^3}K_V(\mathbf{p},\mathbf{p}')\psi_V(\mathbf{p}'), \quad (12)$$

where

$$K_V(\mathbf{p},\mathbf{p}')=K_{V1}(\mathbf{p},\mathbf{p}',m_\phi)+K_{V2}(\mathbf{p},\mathbf{p}',m_\phi)+K_{V3}(\mathbf{p},\mathbf{p}',m_\eta), \quad (13)$$



(a) (b)

FIG. 3. The leading-order interaction diagrams of a bound state composed of a pseudoscalar and a vector by direct (a) and cross (b) ϕ meson exchange.

where the expressions of $K_{V1}(\mathbf{p},\mathbf{p}',m_\phi)$, $K_{V2}(\mathbf{p},\mathbf{p}',m_\phi)$, and $K_{V3}(\mathbf{p},\mathbf{p}',m_\eta)$ can be found in Ref. [28].

C. The bound state (0^+) composed of two vector mesons

The quantum number J^P of the bound state composed of two vector mesons can only be 0^+ or 2^+ since the total wave function should be symmetric. The BS wave function of 0^+ state $|S'\rangle$ is defined as follows:

$$\langle 0|\mathrm{T}\phi_1^\mu(x_1)\phi_2^\nu(x_2)|S'\rangle = \chi_0(x_1,x_2)g^{\mu\nu}. \quad (14)$$

The equation for the BS wave function is given by

$$\chi_0(p) = \frac{1}{4}\Delta_{1\mu\lambda}\int\frac{d^4p'}{(2\pi)^4}K_0^{\alpha\alpha'\mu\mu'}(p,p')\chi_0(p')\Delta_{2\mu'\lambda'}g_{\alpha\alpha'}g^{\lambda\lambda'}, \quad (15)$$

where $\Delta_{j\mu\lambda} = \frac{i}{p_j^2-m_j^2}(\frac{p_{j\mu}p_{j\lambda}}{m_j^2} - g_{\mu\lambda})$.

With the effective interaction Feynman diagrams depicted in Fig. 4, we obtain

$$K_0^{\alpha\alpha'\mu\mu'}(p,p') = \sqrt{2}K_{01}^{\alpha\alpha'\mu\mu'}(p,p',m_\phi) + \sqrt{2}K_{03}^{\alpha\alpha'\mu\mu'}(p,p',m_\eta),$$
$$K_{01}^{\alpha\alpha'\mu\mu'}(p,p',m_V) = iC_{01}\frac{q^\nu q^{\nu'}/m_V{}^2 - g^{\nu\nu'}}{q^2-m_V^2}[g_{D^*D^*V}g^{\alpha\mu}(p_1+p'_1)_\nu - 2g'_{D^*D^*V}(q^\alpha g_{\mu\nu} - q^\mu g_{\alpha\nu})]$$
$$[g_{D^*D^*V}g^{\alpha'\mu'}(p_2+p'_2)_{\nu'} + 2g'_{D^*D^*V}(q^{\alpha'}g_{\mu'\nu'} - q^{\mu'}g_{\alpha'\nu'})]F(q)^2,$$
$$K_{03}^{\alpha\alpha'\mu\mu'}(p,p',m_P) = C_{03}g^2_{D^*D^*P}\varepsilon^{\alpha\beta\mu\nu}q_\nu(p_1+p'_1)_\beta\varepsilon^{\alpha'\beta'\mu'\nu'}q_{\nu'}(p_2+p'_2)_{\beta'}\frac{-i}{q^2-M_\pi^2}F(q)^2. \quad (16)$$

The contribution from vector-meson-exchange (ϕ) is included in $K_{01}^{\alpha\alpha'\mu\mu'}(p,p',m_V)$ and that for exchanging pseudoscalar (η) is included in $K_{03}^{\alpha\alpha'\mu\mu'}(p,p',m_P)$ (the labels $K_{01,03}$ are based on the definition of functions in Ref. [28]). The coefficients C_{01} and C_{03} are 1 and $\frac{2}{3}$, respectively.

Defining $K_0(p,p') = \frac{1}{4}K_0^{\alpha\alpha'\mu\mu'}(p,p')(\frac{p_{2\mu'}p_{2\lambda'}}{m_2^2} - g_{\mu'\lambda'})(\frac{p_{1\mu}p_{1\lambda}}{m_1^2} - g_{\mu\lambda})$, we derive the BS equation that is similar to Eq. (6) but possesses a different kernel.

The BS equation can be reduced to

$$\frac{E^2-(E_1+E_2)^2}{(E_1+E_2)/E_1E_2}\psi_0(\mathbf{p}) = \frac{i}{2}\int\frac{d^3\mathbf{p}'}{(2\pi)^3}K_0(\mathbf{p},\mathbf{p}')\psi_0(\mathbf{p}'), \quad (17)$$

where

$$K_0(\mathbf{p},\mathbf{p}') = \sqrt{2}K_{01}(\mathbf{p},\mathbf{p}',m_\phi) + \sqrt{2}K_{03}(\mathbf{p},\mathbf{p}',m_\eta). \quad (18)$$

The expressions of $K_{01}(\mathbf{p},\mathbf{p}',m_V)$ and $K_{03}(\mathbf{p},\mathbf{p}',m_P)$ also can be found in Ref. [28].

D. The BS equation of 2^+ state $|T'\rangle$ that is composed of two vectors

The BS wave function of the 2^+ state composed of two axial vectors is written as

$$\langle 0|T\phi^\alpha(x_1)\phi^{\alpha'}(x_2)|T'\rangle = \frac{1}{\sqrt{5}}\chi_2(x_1,x_2)\varepsilon^{\alpha\alpha'}, \quad (19)$$

where $\varepsilon^{\alpha\alpha'}$ is the polarization tensor of the 2^+ state. Summation over the polarizations of $\varepsilon^{\alpha\alpha'}\varepsilon^{*\beta\beta'}$ is $(Q^{\alpha\beta}Q^{\alpha'\beta'} + Q^{\alpha\beta'}Q^{\alpha'\beta})/2 - Q^{\alpha\alpha'}Q^{\beta\beta'}/3$ with $Q^{\alpha\beta} = P^\alpha P^\beta/M^2 - g^{\alpha\beta}$.

The BS equation can be expressed as

$$\chi_2(p) = \frac{1}{5}\varepsilon^{\lambda\lambda'}\Delta_{1\mu\lambda}\int\frac{d^4q}{(2\pi)^4}K_2^{\alpha\alpha'\mu\mu'}(p,p')\varepsilon_{\alpha\alpha'}\chi_2(q)\Delta_{2\mu'\lambda'}, \quad (20)$$

where $K_2^{\alpha\alpha'\mu\mu'}(p,p')$ is the same as $K_0^{\alpha\alpha'\mu\mu'}(p,p')$ in Eq. (16).

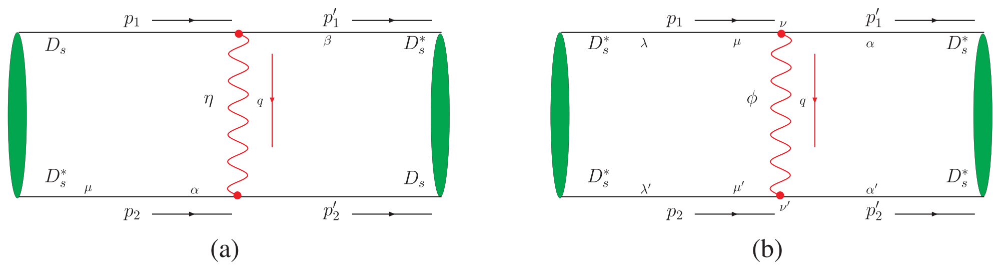


FIG. 4. A bound state composed of two vectors. (a) η is exchanged. (b) ϕ is exchanged.

Defining $K_2(p,p') = \frac{K_2^{\alpha\alpha'\mu\mu'}(p,p')}{5}\varepsilon^{\lambda\lambda'}(\frac{p_{2\mu'}p_{2\lambda'}}{m_2^2} - g_{\mu'\lambda'})(\frac{p_{1\mu}p_{1\lambda}}{m_1^2} - g_{\mu\lambda})\varepsilon_{\alpha\alpha'}$, we reduce the BS equation to the following form:

$$\frac{E^2-(E_1+E_2)^2}{(E_1+E_2)/E_1E_2}\psi_2(\mathbf{p}) = \frac{i}{2}\int\frac{d^3\mathbf{p}'}{(2\pi)^3}K_2(\mathbf{p},\mathbf{p}')\psi_2(\mathbf{p}'), \quad (21)$$

where

$$K_2(\mathbf{p},\mathbf{p}') = \sqrt{2}K_{21}(\mathbf{p},\mathbf{p}',m_\phi) + \sqrt{2}K_{23}(\mathbf{p},\mathbf{p}',m_\eta). \quad (22)$$

The expressions of $K_{21}(\mathbf{p},\mathbf{p}',m_\phi)$ and $K_{23}(\mathbf{p},\mathbf{p}',m_\eta)$ are presented in Ref. [28].

III. NUMERICAL RESULTS

In this section, we solve the BS equations (6), (12), (17), and (21) to study whether these bound states can exist. Since we only focus on the ground state of a bound state, the function $\psi_J(\mathbf{p})$ (J represents S, V, 0, or 2) only depends on the norm of the three-momentum. Therefore, we can first integrate over the azimuthal angle of the function in (6), (12), (17), or (21),

$$\frac{i}{2}\int\frac{d^3\mathbf{p}'}{(2\pi)^3}K_J(\mathbf{p},\mathbf{p}')$$

to obtain a potential form $U_J(|\mathbf{p}|,|\mathbf{p}'|)$. After that, the BS equation turns into a one-dimensional integral equation

$$\psi_J(|\mathbf{p}|) = \frac{(E_1+E_2)/E_1E_2}{E^2-(E_1+E_2)^2}\int d|\mathbf{p}'|U_J(|\mathbf{p}|,|\mathbf{p}'|)\psi_J(|\mathbf{p}'|). \quad (23)$$

When the potential $U_J(\mathbf{p},\mathbf{p}')$ is attractive and strong enough, the corresponding BS equation has one or multiple solutions and we can obtain the spectrum (or spectra) of the possible bound state(s). In general, the standard way of solving such integral equation is to discretize the variable and then perform algebraic operations. The detail of this approach can be found in [28,32,33].

In our calculation, the values of the parameters $g_{D_sD_sV}$, $g_{D_sD_s^*P}$, $g_{D_sD_s^*V}$, $g_{D_s^*D_s^*V}$, and $g'_{D_s^*D_s^*V}$ are presented in the Appendix. In Ref. [28], which suggested T_{cc}^+ is a D^0D^{*+} molecular state, Λ was fixed to be 1.134 GeV. For the $D_s^{(*)}D_s^{(*)}$ and $B_s^{(*)}B_s^{(*)}$ systems, we adjust Λ around that value. The masses of the concerned constituent mesons m_{D_s}, $m_{D_s^*}$, m_{B_s}, and $m_{B_s^*}$ are directly taken from a booklet by the Particle Data Group [46].

A. The results of $D_s^{(*)}D_s^{(*)}$ system

Here we calculate the eigenvalues of bound states $D_sD_s(0^+)$, $D_sD_s^*(1^+)$, $D_s^*D_s^*(0^+)$, and $D_s^*D_s^*(2^+)$, respectively. For fixed parameters $\Lambda = 1.134$ GeV and abovementioned coupling constants, all BS equations are unsolvable, so we try to vary the parameter Λ or coupling constants to search for the solutions of these equations. We can obtain a solution with the binding energy $\Delta E = 1$ MeV when we set $\Lambda = 2.564$ GeV or coupling constants to be 3.261 times the original. It implies the effective interaction between the two constituents is relatively weak. In Table I we list the values of Λ for different bound state of $D^{(*)}D^{(*)}$ where the binding energy ΔE is 1 MeV. We find the value of Λ for the $D_s^*D_s^*$ with $J^P = 0^+$ is closest to the value 1.134 GeV that we fixed in Ref. [28]. In order to study the sensitivity of our results on the choice of the coupling constants that are determined by the flavor SU(3) symmetry, we multiply all coupling constants by 1.2 or 0.8, and the corresponding Λ is marked as $\Lambda_{1.2}$ or $\Lambda_{0.8}$ in Table I, where the ellipses mean the BS equation has no solution. We find that, for bound states $D_sD_s(0^+)$, $D_sD_s^*(1^+)$, and $D_s^*D_s^*(2^+)$, the value of Λ is significantly affected by variations of coupling constants, indicating the instability of solutions. For $D_s^*D_s^*(0^+)$, we find the solution is reasonably stable under the perturbation of coupling constants. To further confirm its stability, we also vary the binding energy of the $D_s^*D_s^*$ state from 0.1 to 2 MeV and the corresponding values of Λ are collected in Table II. We can

TABLE I. The Λ, $\Lambda_{1.2}$, and $\Lambda_{0.8}$ for the ground $D_s^{(*)}D_s^{(*)}$ system with fixed binding energy $\Delta E = 1$ MeV (in units of GeV).

	$D_sD_s(0^+)$	$D_sD_s^*(1^+)$	$D_s^*D_s^*(0^+)$	$D_s^*D_s^*(2^+)$
Λ	2.848	2.564	1.511	13.78
$\Lambda_{1.2}$	2.143	2.056	1.376	3.308
$\Lambda_{0.8}$	7.564	4.068	1.743	…

TABLE II. The values of Λ for the ground $D_s^*D_s^*(0^+)$ system with different binding energies.

ΔE (MeV)	0.1	0.2	0.5	1	1.5	2
Λ (GeV)	1.501	1.502	1.506	1.511	1.516	1.519

TABLE III. The Λ and the binding energy ΔE (in parentheses) for the ground $D_s^{(*)}D_s^{(*)}$ system in references [in units of GeV (MeV)].

	$D_sD_s(0^+)$	$D_sD_s^*(1^+)$	$D_s^*D_s^*(0^+)$	$D_s^*D_s^*(2^+)$
[23]	2.76 (2.43)	2.62 (0.41)	2.76 (2.43)	2.60 (0.86)
[24]	⋯	⋯	⋯	⋯
[26]	⋯	3.40 (0.1)	⋯	3.0 (0.9)

see that Λ is not sensitive to the change of the binding energy.

The possible molecular states of the $D_s^{(*)}D_s^{(*)}$ system have been explored in previous works [23,24,26] and we summarize these results in Table III. Here and below, the ellipses mean there is no solution for the bound state. In Ref. [23], the authors derived the effective potential for the $D_s^{(*)}D_s^{(*)}$ system in the momentum space and computed binding energy of bound states by solving the Schrödinger equation. To compute the effective potential, they employed a one-boson-exchange model, where the Lagrangians are given by chiral perturbation theory with SU(3) flavor symmetry in the heavy quark limit. They suggested that $D_sD_s(0^+)$, $D_sD_s^*(1^+)$, $D_s^*D_s^*(0^+)$, and $D_s^*D_s^*(2^+)$ might not be molecules. In Ref. [24], the author also solved the BS equation in on-shell formalism. To derive the effective Lagrangians, they applied a local hidden gauge approach and SU(4) flavor symmetry (u, d, s, c). They found the interaction of the $D_s^{(*)}D_s^{(*)}$ system is repulsive; hence it is impossible to form bound states. In Ref. [26], the authors suggested it is important to include the contribution of J/ψ exchange in the one-meson-exchange model of $D_s^{(*)}D_s^{(*)}$ systems. They also found $D_sD_s^*(1^+)$ and $D_s^*D_s^*(2^+)$ can form when Λ is larger than 3 GeV. Combing these results (Λ is too large or the interaction of the $D_s^{(*)}D_s^{(*)}$ system is repulsive), we suggest it is unlikely to find a bound state in $D_s^{(*)}D_s^{(*)}$ systems.

B. The results of the $B_s^{(*)}B_s^{(*)}$ system

To solve the BS equation in the $B_s^{(*)}B_s^{(*)}$ system, we define coupling constants by applying flavor SU(3) symmetry and heavy quark limit, where $g_{B_sB_sV}$, $g_{B_sB_s^*P}$, $g_{B_sB_s^*V}$, $g_{B_s^*B_s^*V}$, and $g'_{B_s^*B_s^*V}$ are approximately equal to $g_{D_sD_sV}$, $g_{D_sD_s^*P}$, $g_{D_sD_s^*V}$, $g_{D_s^*D_s^*V}$, and $g'_{D_s^*D_s^*V}$, respectively. We then set the binding energy to be 1 MeV and solve for the eigenvalue of the BS equation by varying the parameters Λ. To test the sensitivity of solutions on coupling constants, we also rescale all coupling constants by a factor of 1.2 and 0.8, and repeat the computation. We summarize the numerical results in Tables IV and V. We find the values of Λ are relatively stable under variations of coupling constants, which suggests the reliability of numerical computations for the $B_s^{(*)}B_s^{(*)}$ system. Additionally, we find that the Λ value of the $B_s^*B_s^*$ system with 0^+ is a little larger than 1.134 GeV. In Refs. [47,48], the authors suggested a relation $\Lambda = m + \alpha\Lambda_{\rm QCD}$, where m is the mass of the exchanged meson, α is a number of $O(1)$, and $\Lambda_{\rm QCD} = 220$ MeV. We also notice that the values of Λ for the $B_s^{(*)}B_s^{(*)}$ system (see Table VI) are smaller than those for the $D_s^{(*)}D_s^{(*)}$ system (see Table III) in Refs. [23,26] if they exist. Overall, our results suggest that two B_s^* mesons perhaps can form a loose bound state with $J^P = 0^+$.

TABLE IV. The Λ, $\Lambda_{1.2}$, and $\Lambda_{0.8}$ for the ground $B_s^{(*)}B_s^{(*)}$ system with fixed binding energy $\Delta E = 1$ MeV (in units of GeV).

	$B_sB_s(0^+)$	$B_sB_s^*(1^+)$	$B_s^*B_s^*(0^+)$	$B_s^*B_s^*(2^+)$
Λ	1.677	1.653	1.293	2.231
$\Lambda_{1.2}$	1.666	1.505	1.179	1.852
$\Lambda_{0.8}$	2.574	1.929	1.483	3.532

The possible bound states of the $B_s^{(*)}B_s^{(*)}$ system have been studied based on different phenomenological models [23,25,26]. Their results are listed in Table VI. In Ref. [23], the authors suggested $B_sB_s(0^+)$, $B_sB_s^*(1^+)$, $B_s^*B_s^*(0^+)$, and $B_s^*B_s^*(2^+)$ might be candidates of molecular states (Table VI). The corresponding Λ and binding energies are approximately consistent with our results. In Ref. [26], the authors also explored these systems in a quasipotential BS equation approach. They found that only $B_sB_s^*(1^+)$ and $B_s^*B_s^*(2^+)$ can form molecular states. Their deviation from our results may be due to the difference in interaction kernels, where they also included the contributions of Υ meson exchange. Another potential source of difference may come from different approximations in solving the BS equation. Future study is required to compare numerical precision of these approximations. In Ref. [25], the authors claimed bound states cannot exist because the interaction between $B_s^{(*)}$ and $B_s^{(*)}$ is repulsive. The discrepancy between their results and ours may be due to the different

TABLE V. The value of Λ for the ground $B_s^*B_s^*(0^+)$ system with varying binding energies ΔE.

ΔE (MeV)	0.1	0.2	0.5	1	1.5	2
Λ (GeV)	1.273	1.276	1.284	1.293	1.301	1.308

TABLE VI. The Λ and the binding energy ΔE (in the parentheses) for the ground $B_s^{(*)}B_s^{(*)}$ system in references [in units of GeV (MeV)].

	$B_sB_s(0^+)$	$B_sB_s^*(1^+)$	$B_s^*B_s^*(0^+)$	$B_s^*B_s^*(2^+)$
[23]	1.90 (2.27)	1.82 (0.83)	1.90 (2.27)	1.82 (0.22)
[25]	⋯	⋯	⋯	⋯
[26]	⋯	2.84 (1.5)	⋯	2.1 (0.2)

forms of effective interactions used in the BS equation. In summary, so far there is no consensus on possible bound states in the $B_s^{(*)}B_s^{(*)}$ system, and future explorations in both theory and experiment will be crucial to elucidate the nature of the $B_s^{(*)}B_s^{(*)}$ system.

IV. A BRIEF SUMMARY

Following the approach in our recent work, we study whether two $D_s^{(*)}$ or $B_s^{(*)}$ mesons can form a hadronic molecule within the BS framework. In Refs. [32–36], the BS equation was applied to the systems of one particle and one antiparticle. In Ref. [28], we studied the possible bound states of a particle-particle system (two $D^{(*)}$ or $B^{(*)}$ mesons). In this work, we extend this framework to the $D_s^{(*)}D_s^{(*)}$ or $B_s^{(*)}B_s^{(*)}$ systems and compute the binding energy of such hadronic molecules by solving BS equations.

We employ heavy meson chiral perturbation theory and leading-order (one-boson-exchange) approximation to calculate the interaction kernels of the BS equations for the $D_s^{(*)}D_s^{(*)}$ or $B_s^{(*)}B_s^{(*)}$ systems, where η or ϕ is exchanged. All coupling constants are taken from relevant references. Adopting the fixed value of Λ from the analysis of T_{cc}^+ under the hypothesis that T_{cc}^+ is a bound state of D^0D^{*+} with $I=0$ and $J=1$, we find all these BS equations have no solution. Then we set the binding energy $\Delta E = 1$ MeV and test different values of Λ or coupling constants for $D_s^{(*)}D_s^{(*)}$ or $B_s^{(*)}B_s^{(*)}$ systems with defined quantum number. We find that, for the $D_s^{(*)}D_s^{(*)}$ system, a larger Λ or coupling constants are needed to form bound states. In light of previous studies [23,24,26] and our results, we suggest the chance of forming molecular states of $D_s^{(*)}D_s^{(*)}$ is low. For the $B_s^*B_s^*$ system, we find our results are reasonably stable under variation of parameters and roughly agree with Ref. [23]. It is noted that, in our computations, the Λ values of $B_s^*B_s^*$ with $J^P=0^+$ is close to that fixed from T_{cc}^+, which supports the existence of such a molecular bound state. Overall, our results suggest that $B_s^*B_s^*$ might form a 0^+ molecular state. We also notice there are still some discrepancies on possible molecular states of $B_s^{(*)}B_s^{(*)}$ [23,25,26]. These may come from different approximations in solving BS equations or different forms of the effective Lagrangians in the models. We admit that, in our calculations, both the approximation of BS equations and estimation of model parameters can potentially cause errors in numerical results. However, the major goal of this work is to qualitatively analyze whether two $D_s^{(*)}$ or $B_s^{(*)}$ mesons can form a molecular state. Even if the numerical estimation is not perfectly precise, our result can still provide a useful guidance for the future study of the double charmed hadronic molecular state. Eventually, further theoretical and experimental works are needed for gaining a better understanding of the double charmed/bottomed exotic states.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (NNSFC) under Contract No. 12075167.

APPENDIX: THE EFFECTIVE INTERACTIONS

The effective interactions can be found in [40–42],

$$\mathcal{L}_{DDV} = g_{DDV}(D_b \overleftrightarrow{\partial}_\beta D_a^\dagger)(\mathcal{V}^\beta)_{ba}, \tag{A1}$$

$$\mathcal{L}_{DD^*V} = i g_{DD^*V}\varepsilon^{\alpha\beta\mu\nu}(\partial_\alpha \mathcal{V}_\beta - \partial_\beta \mathcal{V}_\alpha)_{ba} (\partial_\nu D_b D_a^{*\mu\dagger} - \partial_\nu D_b^{*\mu\dagger} D_a), \tag{A2}$$

$$\mathcal{L}_{DD^*P} = g_{DD^*P} D_b(\partial_\mu \mathcal{M})_{ba} D_a^{*\mu\dagger} + g_{DD^*P} D_b^{*\mu}(\partial_\mu \mathcal{M})_{ba} D_a^\dagger, \tag{A3}$$

$$\mathcal{L}_{D^*D^*P} = g_{D^*D^*P}(D_b^{*\mu} \overleftrightarrow{\partial}^\beta D_a^{*\alpha\dagger})(\partial^\nu \mathcal{M})_{ba}\varepsilon_{\nu\mu\alpha\beta}, \tag{A4}$$

$$\mathcal{L}_{D^*D^*V} = i g_{D^*D^*V}(D_b^{*\nu} \overleftrightarrow{\partial}_\mu D_{a\nu}^{*\dagger})(\mathcal{V})^\mu_{ba} + i g'_{D^*D^*V}(D_b^{*\mu} D_a^{*\nu\dagger} - D_b^{*\mu\dagger} D_a^{*\nu})(\partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu)_{ba}, \tag{A5}$$

where a and b represent the index of the SU(3) flavor group for three light quarks. In Ref. [40], $\mathcal{M}$ and $\mathcal{V}$ are 3×3 Hermitian and traceless matrices

$$\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

and

$$\begin{pmatrix} \frac{\rho^0}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix},$$

respectively.

In the flavor SU(3) symmetry and heavy quark limit, the above coupling constants are given by $g_{D_sD_sV}=\frac{\beta g_V}{\sqrt{2}}$, $g_{D_sD_s^*V}=\frac{\lambda g_V}{\sqrt{2}}$, $g_{D_s^*D_s^*P}=\frac{g}{f_\pi}$, $g_{D_sD_s^*P}=g_{DD^*P}=-\frac{2g}{f_\pi}\sqrt{M_DM_{D^*}}$, $g_{D_s^*D_s^*V}=-\frac{\beta g_V}{\sqrt{2}}$, $g'_{D_s^*D_s^*V}=g'_{D^*D^*V}=-\sqrt{2}\lambda g_V M_{D^*}$ with $f_\pi=132$ MeV [41], $g=0.64$ [42], $\kappa=g$, $\beta=0.9$, $g_V=5.9$ [49], and $\lambda=0.56\ \mathrm{GeV}^{-1}$ [50].

[1] R. Aaij *et al.* (LHCb Collaboration), arXiv:2109.01056.
[2] R. Aaij *et al.* (LHCb Collaboration), arXiv:2109.01038.
[3] L. Meng, G. J. Wang, B. Wang, and S. L. Zhu, Phys. Rev. D **104**, 051502 (2021).
[4] M. J. Yan and M. P. Valderrama, Phys. Rev. D **105**, 014007 (2022).
[5] H. Ren, F. Wu, and R. Zhu, Adv. High Energy Phys. **2022**, 9103031 (2022).
[6] M. L. Du, V. Baru, X. K. Dong, A. Filin, F. K. Guo, C. Hanhart, A. Nefediev, J. Nieves, and Q. Wang, Phys. Rev. D **105**, 014024 (2022).
[7] L. R. Dai, R. Molina, and E. Oset, Phys. Rev. D **105**, 016029 (2022).
[8] Q. Xin and Z. G. Wang, arXiv:2108.12597.
[9] A. Feijoo, W. H. Liang, and E. Oset, Phys. Rev. D **104**, 114015 (2021).
[10] C. Deng and S. L. Zhu, Phys. Rev. D **105**, 054015 (2022).
[11] K. Chen, R. Chen, L. Meng, B. Wang, and S. L. Zhu, arXiv:2109.13057.
[12] S. S. Agaev, K. Azizi, and H. Sundu, Nucl. Phys. **B975**, 115650 (2022).
[13] M. J. Zhao, Z. Y. Wang, C. Wang, and X. H. Guo, Phys. Rev. D **105**, 096016 (2022).
[14] M. Z. Liu, T. W. Wu, M. Pavon Valderrama, J. J. Xie, and L. S. Geng, Phys. Rev. D **99**, 094018 (2019).
[15] U. Özdem, Phys. Rev. D **105**, 054019 (2022).
[16] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003).
[17] K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **98**, 082001 (2007).
[18] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **94**, 182002 (2005).
[19] S. K. Choi *et al.* (BELLE Collaboration), Phys. Rev. Lett. **100**, 142001 (2008).
[20] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **127**, 082001 (2021).
[21] I. Adachi *et al.* (Belle Collaboration), arXiv:1105.4583.
[22] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **122**, 222001 (2019).
[23] N. Li, Z. F. Sun, X. Liu, and S. L. Zhu, Phys. Rev. D **88**, 114008 (2013).
[24] R. Molina, T. Branz, and E. Oset, Phys. Rev. D **82**, 014010 (2010).
[25] L. R. Dai, E. Oset, A. Feijoo, R. Molina, L. Roca, A. M. Torres, and K. P. Khemchandani, Phys. Rev. D **105**, 074017 (2022).
[26] Z. M. Ding, H. Y. Jiang, D. Song, and J. He, Eur. Phys. J. C **81**, 732 (2021).
[27] X. Y. Gao *et al.* (Belle Collaboration), Phys. Rev. D **105**, 032002 (2022).
[28] H. W. Ke, X. H. Liu, and X. Q. Li, Eur. Phys. J. C **82**, 144 (2022).
[29] X. H. Guo, A. W. Thomas, and A. G. Williams, Phys. Rev. D **59**, 116007 (1999).
[30] Q. Li, C. H. Chang, S. X. Qin, and G. L. Wang, Chin. Phys. C **44**, 013102 (2020).
[31] M.-H. Weng, X.-H. Guo, and A. W. Thomas, Phys. Rev. D **83**, 056006 (2011).
[32] H. W. Ke, X. H. Liu, and X. Q. Li, Chin. Phys. C **44**, 093104 (2020).
[33] H. W. Ke, X. Q. Li, Y. L. Shi, G. L. Wang, and X. H. Yuan, J. High Energy Phys. 04 (2012) 056.
[34] H. W. Ke, M. Li, X. H. Liu, and X. Q. Li, Phys. Rev. D **101**, 014024 (2020).
[35] X. H. Guo and X. H. Wu, Phys. Rev. D **76**, 056004 (2007).
[36] G. Q. Feng, Z. X. Xie, and X. H. Guo, Phys. Rev. D **83**, 016003 (2011).
[37] G. Q. Feng and X. H. Guo, Phys. Rev. D **86**, 036004 (2012).
[38] H. W. Ke and X. Q. Li, Eur. Phys. J. C **78**, 364 (2018).
[39] H. W. Ke, X. Han, X. H. Liu, and Y. L. Shi, Eur. Phys. J. C **81**, 427 (2021).
[40] G. J. Ding, Phys. Rev. D **79**, 014001 (2009).
[41] P. Colangelo, F. De Fazio, and R. Ferrandes, Phys. Lett. B **634**, 235 (2006).
[42] P. Colangelo, F. De Fazio, F. Giannuzzi, and S. Nicotri, Phys. Rev. D **86**, 054024 (2012).
[43] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281**, 145 (1997).
[44] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Lett. B **292**, 371 (1992).
[45] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Lett. B **299**, 139 (1993).
[46] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).
[47] C. Meng and K. T. Chao, Phys. Rev. D **77**, 074003 (2008).
[48] H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D **71**, 014030 (2005).
[49] A. F. Falk and M. E. Luke, Phys. Lett. B **292**, 119 (1992).
[50] R. Chen, Z. F. Sun, X. Liu, and S. L. Zhu, Phys. Rev. D **100**, 011502 (2019).